Skew Symmetric Distributions

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Abstract

Since Azzalini (1985,1986), the univariate and multivariate skew-normal distributions have been investigated by many authors. This class of distributions includes the normal and has some properties like the normal and yet is skew. In this work, more general skew-symmetric distributions are investigated. For the univariate case, a random variable has generalized skew-symmetric distribution, if and only if its probability density function (p.d.f.) has the form $2f(x)G(x),\ x \in \mathbb{R}$, where G is a skew function, that is G is a Lebesgue measurable function satisfying $0 \leq G(x) \leq 1$ and $G(x)+G(-x) = 1$ a.e. on $\mathbb{R}$. In particular, properties of generalized skew-normal, generalized skew-Cauchy, and generalized skew-t distributions are given. For the multivariate case, among others, we study the multivariate skew normal-symmetric distributions with p.d.f. of the form $f_{\boldsymbol{Z}}(\boldsymbol{z}) = 2\phi_p(\boldsymbol{z};\boldsymbol{\Omega})G(\boldsymbol{\alpha}'\boldsymbol{z})$, where $\boldsymbol{\Omega} > 0,\ \boldsymbol{\alpha} \in \mathbb{R}^p$, $\phi_p(\boldsymbol{z};\boldsymbol{\Omega})$ is the p-dimensional normal p.d.f. with zero mean vector and correlation matrix $\boldsymbol{\Omega}$, and G is a skew function. We show certain quadratic forms have parallel properties as the multivariate normal distribution.

Keywords: Quadratic form, skew-Cauchy distribution, skew-normal distribution, skew-symmetric distribution, skew-t distribution.