A Note on Watson's Paper

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Watson in his paper [1] mentions on page 381 that " It is hard to be sure than an increase in α increases $E(M_R)$ but the lower bound to $E(M_R)$ certainly increases." In this short note, the author intends to remove some kind of doubt left in the first part of the statement. Following the notations in [1], we have

$$E(M_R) = kp' \sum_{n=0}^{g} n\pi_2 \left(\sqrt{\frac{nk+1}{2}}\frac{\Delta}{\sigma}, \beta \right) \binom{g}{n} \pi_1^{*n} (1-\pi_1^*)^{g-n} . \qquad (1)$$

For shortness, we shall write $b(n, \pi_1^*, g)$ to denote $\binom{g}{n} \pi_1^{*n} (1-\pi_1^*)^{g-n}$.
Differentiating (1) w.r.t. α, we get

$$\frac{\partial}{\partial\alpha} E(M_R) = k \frac{dp'}{d\alpha} \sum_{n=0}^{g} n\pi_2 \left(\sqrt{\frac{nk+1}{2}}\frac{\Delta}{\sigma}, \beta \right) b(n, \pi_1^*, g)$$

$$+ kp' \sum_{n=0}^{g} (n-\pi_1^* g)\pi_2 \left(\sqrt{\frac{nk+1}{2}}\frac{\Delta}{\sigma}, \beta \right) \frac{d\pi_1^*}{d\alpha} b(n-1, \pi_1^*, g-1)/1-\pi_1^* \quad (2)$$

With slight adjustment, (2) becomes

$$\frac{\partial}{\partial\alpha} E(M_R) = f\pi_1^* \frac{dp'}{d\alpha} \sum_{n=1}^{g} \pi_2 \left(\sqrt{\frac{nk+1}{2}} \frac{\Delta}{\sigma}, \beta \right) b(n-1, \pi_1^*, g-1)$$

$$+ fp' \frac{d\pi_1^*}{d\alpha} \sum_{n=1}^{g} \pi_2 \left(\sqrt{\frac{nk+1}{2}} \frac{\Delta}{\sigma}, \beta \right) b(n-1, \pi_1^*, g-1)$$

$$+ fp' \frac{d\pi_1^*}{d\alpha} \sum_{n=1}^{g} \pi_2 \left(\sqrt{\frac{nk+1}{2}} \frac{\Delta}{\sigma}, \beta \right) \left[(n-1) - (g-1) \pi_1^* \right]$$

$$X\ b(n-1, \pi_1^*, g-1)/\ 1-\pi_1^* . \qquad (3)$$

Next, $p' = p\pi_1'/\pi_1^*$ [cf. (4.14), page 379, [1]]. Therefore,

$$\frac{d}{d\alpha} (\pi_1^* p') = \pi_1^* \frac{dp'}{d\alpha} + p' \frac{d\pi_1^*}{d\alpha} = \frac{d}{d\alpha} (p\pi_1') = p \frac{d\pi_1'}{d\alpha} . \qquad (4)$$

Using this, (3) reduces to

$$\frac{\partial}{\partial\alpha} E(M_R) = fp \frac{d\pi_1'}{d\alpha} \sum_{n=1}^{g} \pi_2 \left(\sqrt{\frac{nk+1}{2}} \frac{\Delta}{\sigma}, \beta \right) b(n-1, \pi_1^*, g-1)$$

$$+ fp' \frac{d\pi_1'}{d\alpha} \sum_{n=1}^{g} \pi_2 \left(\sqrt{\frac{nk+1}{2}} \frac{\Delta}{\sigma}, \beta \right) \left[(n-1) - (g-1) \pi_1^* \right]$$

$$X\ b(n-1, \pi_1^*, g-1)/\ 1-\pi_1^* . \qquad (5)$$

The first term in (5) is obviously positive for a non-zero β and with the same restriction on β, the second term is positive since it is the covari between two non-decreasing functions of n, namely,

$$(n-1) \text{ and } \pi_2\left(\sqrt{\frac{nk+1}{2}}\ \frac{\Delta}{\sigma}, \beta\right)$$

multiplied by a positive term $fp' \frac{d\pi_1^*}{d\alpha} / 1-\pi_1^*$. Hence,

$$\frac{\partial}{\partial\alpha} E(M_R) > 0.$$

Therefore, it is clear that $E(M_R)$ increases as α increases for a given $\beta \neq 0$.

Reference

[1] Watson, G. S., " A Study of the Group Screening Method," Technometrics, Vol. 3, No. 3, August 1961., pp 371-388.