81

Some Aspects of Selection and Ranking Procedures with Applications*

by

Shanti S. Gupta and William J. Studden

Department of Statistics

Division of Mathematical Sciences

Mimeograph Series Number 81

July 1966

* This research was supported in part by the Office of Naval Research Contract NONR 1100(26) and the Aerospace Research Laboratories Contract AF 33(657)11737 at Purdue University. Reproduction in whole or in part is permitted for any purposes of the United States Government.

Some Aspects of Selection and Ranking Procedures

with Applications*

by

Shanti S. Gupta and William J. Studden

Purdue University

1. Introduction and Summary

The purpose of this paper is to discuss the problems of ranking and selection in a more or less unified form and suggest some applications to problems in reliability. More specifically, we discuss the problem of selecting a subset of k populations or processes which in some sense is a best subset.

We are given k populations $\Pi_1, \Pi_2, \ldots, \Pi_k$ with densities $f_{\theta_1}(x)$, $f_{\theta_2}(x)$, ..., $f_{\theta_k}(x)$. We wish to define a rule for selecting a subset of the populations on the basis of observations from each of the k populations. In general, the parameters θ_i are not known and usually range over some subset of the real line. It will be assumed for purposes of our discussion that the larger the parameter θ, the more preferable is the selection of the corresponding population. The population with the largest parameter will be called the best. The definition of the best population is arbitrary, for example, the best could be the smallest. Modifications necessary in most cases will be clear.

The selection of any subset containing the best population is called a correct selection and will be denoted by (CS). If the selection proceeds according to some rule R, then the subset selected should contain the best population with a specified probability P^* i.e.

---

* This research was supported in part by the Office of Naval Research Contract NONR 1100(26) and the Aerospace Research Laboratories Contract AF 33(657)11737 at Purdue University. Reproduction in whole or in part is permitted for any purposes of the United States Government.

(1.1) $$P\{CS|R\} \geq P^* \quad ,$$

whatever the unknown values of θ_i's may be. Moreover, the selection rule R should possess certain desirable properties. For example, the size of the selected subset should be made small in some sense.

In many problems the parameter θ is directly related to some measure of the reliability of the populations or processes. For example, θ may be the mean or median. In other situations one may be interested in some functional of the process, for example, the failure or hazard rate $H(t) = f(t)/(1-F(t))$. In the latter circumstance we may want to select those processes or populations for which $H(t)$ or possibly some functional of $H(t)$ is small.

It should be pointed out that we are considering procedures which select subsets of random size. Other types of procedures have been proposed. These refer mainly to the "indifference zone" type where usually one population is selected. Details of this type of approach for the normal means problem can be found in Bechhofer (1954). Other references can be found in Gupta (1965).

Section 2 of this paper deals with the selection rules for three specific situations. Properties of these rules are discussed in Section 3 and applications and examples are given in Section 4.

2. Some Selection Rules

In this section, we discuss some selection rules when the populations have densities $f_\theta(x)$ of the following form.

(i) $f_\theta(x) = f(x-\theta)$ i.e. θ is a translation parameter.

(ii) $f_\theta(x) = \frac{1}{\theta} f(\frac{x}{\theta})$ i.e. θ is a scale parameter.

(iii) $f_\theta(x) = \sum_{j=0}^{\infty} \frac{e^{-\theta}\,\theta^j}{j!}\, g_j(x)$, where $g_j(x)$, $j = 0,1,\ldots$

is a sequence of density functions on $[0,\infty)$.

Case (iii) arises in certain multivariate problems. The sequence $g_j(x)$ are gamma or F (central) densities so that the resulting mixture $f_\theta(x)$ is either a non-central χ^2 or a non-central F density.

In the general situation we are given an observation x_i from the population Π_i which has density $f_{\theta_i}(x)$. Let the ordered θ_i's be denoted by

(2.1) $$\theta_{[1]} \leq \theta_{[2]} \leq \cdots \leq \theta_{[k]} .$$

The correct pairing of the ordered θ_i's and the observed x_i's is not known. A selection rule consists of k functions $\varphi_i(x_1,x_2,\ldots,x_k)$, $i = 1,2,\ldots,k$ where $\varphi_i(x_1,x_2,\ldots, x_k)$ denotes the probability of selecting the ith population. Usually, the functions φ_i are indicator functions. Now we discuss the three cases separately.

Case (i).

For this case the following rule R has been proposed;

Select Π_i iff

(2.2) $$x_i \geq x_{max} - d$$

where $x_{max} = \max(x_1,x_2,\ldots, x_k)$ and the constant $d \geq 0$ is the minimal value such that $P\{CS|R\} \geq P^*$ $(1/k < P^* < 1)$. The function φ_i in this case is one if (2.2) holds and zero otherwise. The rule R selects a non-empty subset of random size and is translation invariant. Further properties of the rule R are given in the next section.

Cases (ii) and (iii).

In both of these situations the rule given in Case (i) is modified as follows:

Select Π_i iff

$$x_i \geq c\, x_{max} \tag{2.3}$$

where $0 < c < 1$ and c is again chosen to satisfy the P^* condition. It should be noted that the above rule is scale invariant.

It is worthwhile pointing out that the rules given in (2.2) and (2.3) are special cases of the class of rules which select Π_i iff

$$h_b(x_i) \geq x_{max} \tag{2.4}$$

where the functions $h_b(x)$ satisfy certain properties. In case (i), $h_b(x) = x + b$ while in cases (ii) and (iii) $h_b(x) = b\, x$. The value b is chosen to satisfy the P^* condition. In general, we require certain minimal conditions on the function $h_b(x)$. For example, it would be necessary to have $h_b(x) \geq x$ in order that the selected subset be non-empty.

3. Properties of the Selection Rules

In this section, we discuss some properties and performance characteristics of the selection rules given in Section 2. These properties together with numerical evaluations of the performance of these rules provide some justification for their use. The discussion here is mainly concerned with the translation case; the scale parameter case is entirely similar. The modifications and exceptions for case (iii) will be mentioned briefly at the end of this section.

Probability of a Correct Selection

For the rule R defined by (2.2) it is easily seen that

$$P\{CS|R\} = \int_{-\infty}^{\infty} \left[\prod_{i=1}^{k-1} F_{\theta_{[i]}}(x+d)\right] f_{\theta_{[k]}}(x)dx \quad . \tag{3.1}$$

For any distribution function which is stochastically increasing in the sense that $F_\theta(x) \leq F_{\theta'}(x)$ for all $\theta' < \theta$ and all x, we see from (3.1) that

$$P\{CS|R\} \geq \int_{-\infty}^{\infty} \left[F_{\theta_{[k]}}(x+d)\right]^{k-1} f_{\theta_{[k]}}(x)\ dx \ . \tag{3.2}$$

In the translation case, which is obviously stochastically increasing, the right hand side (3.2) is independent of $\theta_{[k]}$ and hence the constant d can be evaluated from the equation

$$\int_{-\infty}^{\infty} F^{k-1}(x+d)\ f(x)\ dx = P^* \tag{3.3}$$

where F and f are independent of the translation parameter.

In the case of translation (3.1) can be rewritten as

$$P\{CS|R\} = \int_{-\infty}^{\infty} \left[\prod_{j=1}^{k-1} F(x+d+\theta_{[k]}-\theta_{[j]})\right] f(x)\ dx \tag{3.4}$$

which shows that the probability of a correct selection using the rule R is an increasing function of each of the differences $\theta_{[k]}-\theta_{[j]}$, $j = 1,2,\ldots, k-1$.

In the more general situation we are given a value P^* in $(1/k,1)$ and require a rule for which the probability of a correct selection is at least equal to P^* for all configurations of the parameters. Thus we essentially require a sharp lower bound on an expression of the type given in (3.1). This lower bound is then set equal to P^* to solve for the constant which defines a specific selection rule. It would seem that in obtaining such a lower bound the first step for any "reasonable" selection rule would be to set all the θ_i values equal to some fixed θ as we did in going from (3.1) to (3.2). In this sense the set of parameter values where all the θ_i's are equal is "worst" or "least favorable" configuration.

Monotonicity and Unbiasedness

Using the same argument as in (3.4), one can obtain

(3.5) $$P\{\text{Selecting the population corresponding to } \theta_{[i]} | R\}$$

$$= \int_{-\infty}^{\infty} \Big[\prod_{\substack{\ell=1 \\ \ell \neq i}}^{k} F(x+d+\theta_{[i]}-\theta_{[\ell]}) \Big] f(x)\, dx \quad .$$

From (3.5), we deduce that for the rule R the probability of selecting the population corresponding to $\theta_{[i]}$ is greater than or equal to the probability of selecting the population corresponding to $\theta_{[j]}$ provided $\theta_{[i]} \geq \theta_{[j]}$. Thus, the rule R is unbiased in the sense that the probability of rejecting any population not having the largest parameter θ is not less than the probability of rejecting the best population.

Expected Size of the Selected Subset

In trying to obtain selection rules which are optimal in some sense a number of natural criteria arise. For example one might try to find a rule satisfying the P^* condition of (1.1) which minimizes the expected size of the selected subset, the expected sum of ranks of the selected populations or some other appropriate expression. This minimization could be attempted uniformly for all configurations of the θ_i values or for a restricted set of θ_i values if no uniformly best rule exists. We shall confine our remarks here to the subset size which we denote by S.

It can be shown that the rule R defined in (2.2) has the property that if we set the first m of the θ values equal; i.e. we let $\theta_{[1]} = \theta_{[2]} = \dots = \theta_{[m]} = \theta \leq \theta_{[m+1]} \leq \dots \leq \theta_{[k]}$ then $E(S|\theta_{[1]}=\dots=\theta_{[m]}=\theta)$ is a non-decreasing function of θ for $\theta \leq \theta_{[m+1]}$. Thus the expected subset size increases if we set all of the θ_i values equal to $\theta_{[k]}$. It then follows, since $ES = \sum_{i=1}^{k} P(\text{selecting } \Pi_i)$, that

$$\sup_{\Omega} ES \leq k \int F^{k-1}(x+d) \; f(x)\, dx$$
$$= k P^* .$$

(The supremum is taken over the set Ω of all configurations of the θ_i values.)

Thus subject to the basic P* requirement the procedure R satisfies the condition that the expected size of the selected subset is $\leq k P^*$ for all choices of $\theta_1, \theta_2, \dots, \theta_k$.

Minimax Property of the Rule R.

Let $\underline{\theta} = (\theta_1, \dots, \theta_k)$ denote the vector of parameters associated with the populations $\Pi_1, \dots, \Pi_k$. It will be noticed that the rule R defined in (2.2) satisfies the equations

$$\text{(3.6)} \qquad \inf_{\Omega} P_{\underline{\theta}}(CS|R) = P_{\underline{\theta}_0}(CS|R) = P^*$$

and

$$(3.7) \qquad \sup_{\Omega} E_{\underline{\theta}} (S|R) = E_{\underline{\theta}_0} (S|R)$$

where $\underline{\theta}_0 = (\theta_0, \theta_0, \ldots, \theta_0)$ is some vector with all components equal. For notational convenience we have now displayed $\underline{\theta}$ which was previously suppressed. For the translation case we note, in fact, that the right sides of (3.6) and (3.7) are actually independent of θ_0.

We now show that equations (3.6) and (3.7), together with a simple invariance condition, imply that the rule R is minimax in the sense described below.

Suppose that $x_1, \ldots, x_k$ are a set of observations from the k populations $\Pi_1, \ldots, \Pi_k$ and that with this set of observations we select the ith population with probability $\varphi_i(x_1, \ldots, x_k)$. The invariance or symmetry condition we impose is that if the ith and jth observations are interchanged, i.e. x_j is observed from Π_i and x_i from Π_j, then we select the jth population with the same probability $\varphi_i(x_1, \ldots, x_k)$. More specifically, we shall require that

$$(3.8) \qquad \varphi_i(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_k) = \varphi_j(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_k)$$

for all i and j.

A simple change of variable shows that for any rule R' satisfying (3.8) we have

$$(3.9) \qquad E_{\underline{\theta}_0}(S|R') = \sum_{i=1}^{k} P_{\underline{\theta}_0} (\text{selecting } \Pi_i | R')$$

$$= \sum_{i=1}^{k} \int \varphi_i(x_1, \ldots, x_k) \left[\prod_{j=1}^{k} f_{\theta_0}(x_j) \right] dx_1, \ldots, dx_k$$

$$= k\, P_{\underline{\theta}_0} (CS|R').$$

It should be noted that in the case of two or more populations having equal values $\theta_{[k]}$, one of them is considered ''tagged'' and a correct selection is defined as the selection of any subset that includes this ''tagged'' population. From (3.9)

$$(3.10) \qquad k[P_{\underline{\theta}_0}(CS|R')-P_{\underline{\theta}_0}(CS|R)] = E_{\underline{\theta}_0}(S|R')-E_{\underline{\theta}_0}(S|R).$$

If R'. satisfies the basic P* condition it follows from (3.6) that the left side of (3.10) is nonnegative. This fact together with (3.7) implies that

$$E_{\underline{\theta}_0}(S|R') \geq E_{\underline{\theta}_0}(S|R) = \sup_{\Omega} E_{\underline{\theta}}(S|R)$$

so that

$$(3.11) \qquad \sup_{\Omega} E_{\underline{\theta}}(S|R') \geq \sup_{\Omega} E_{\underline{\theta}}(S|R).$$

<u>Thus the rule R is minimax in the sense that it minimizes $\sup_{\Omega} E_{\underline{\theta}}(S|R')$ over the class of rules satisfying the basic P* condition and the invariance condition (3.8)</u>.

<u>Some Comments on Case (iii)</u>.

The properties we have described in this section for the translation parameter also hold for the scale parameter case. However in case (iii) several changes should be noted. In the following all of the expressions will be calculated using the ''scale'' rule proposed for case (iii) and Ω_0 will denote that subset of Ω for which all of the parameter components are equal.

We first observe that if the distribution function $F_\theta(x)$ is stochastically increasing then

$$(3.12) \qquad \inf_{\Omega} P_{\underline{\theta}}(CS) = \inf_{\Omega_0} P_{\underline{\theta}}(CS).$$

In case (iii) however the probability of correct selection when all of the θ_i values are equal to θ is not independent of θ. Sufficient conditions are given in Gupta and Studden (1965) which guarantee that the infimum on the right side of (3.12) will occur when all θ_i are equal to zero.

The statements concerning the monotonicity and unbiasedness for the translation and scale case hold whenever $f_\theta(x)$ has a monotone likelihood ratio. This is the situation in case (iii) if $g_j(x)$, $j=0,1,\ldots,$ represents central χ^2 or central F densities.

The situation when considering the expected size of the selected subset is similar to that described above for the probability of a correct selection, i.e.,

$$\sup_{\Omega} E_{\underline{\theta}} S = \sup_{\Omega_0} E_{\underline{\theta}} S.$$

It can also be shown that for case (iii) the supremum on the right side in the above expression is k and not kP^* as in cases (i) and (ii). The value k is obtained as the common value θ approaches infinity. The above remarks concerning ES were also noted by Alam and Rizvi (1965).

The fact that $\sup_{\Omega} E_{\underline{\theta}} S = k$ implies that the rule proposed for case (iii) is not minimax since the randomized rule which selects each population with probability P^* satisfies $E_{\underline{\theta}} S \equiv kP^*$.

4. Applications and Examples

Selection and Ranking of Multivariate Normal Populations in terms of $\lambda_i = \underline{\mu}_i' \Sigma_i^{-1} \underline{\mu}_i$.

Let Π_i: $N(\underline{\mu}_i, \Sigma_i)$, $i = 1,2,\ldots, k$ be p-variate normal populations with mean vectors $\underline{\mu}_i$ and covariance matrices Σ_i, respectively. Let $\lambda_i = \underline{\mu}_i' \Sigma_i^{-1} \underline{\mu}_i$. Assume $\Sigma_1, \Sigma_2, \ldots, \Sigma_k$ are known. Let $\underline{x}_{ij}$ denote the jth observation of the p-dimensional random vector on the ith population. We have $j = 1,2,\ldots,n$ independent observations from each of the k populations and we compute

$$(4.1) \qquad y_{ij} = \underline{x}_{ij}' \Sigma_i^{-1} \underline{x}_{ij}, \qquad i = 1,2,\ldots, k; \quad j = 1,2,\ldots, n \quad .$$

Since y_{ij}, $j = 1,2,\ldots, n$ correspond to n independent observations on a non-central χ^2 for each i, then $Y_i = \sum_{j=1}^{n} Y_{ij}$ is distributed as a non-central χ^2 with non-centrality parameter $\lambda_i' = n\lambda_i = n\underline{\mu}_i' \Sigma_i^{-1} \underline{\mu}_i$ and degrees of freedom $p' = np$. The proposed selection rule for the population with the largest value of λ_i is:

select Π_i iff

$$(4.2) \qquad c \sum_{j=1}^{n} y_{ij} \geq \max_i \left\{ \sum_{j=1}^{n} y_{ij}, \quad i = 1,2,\ldots, k \right\}$$

where the constant $c = c(k,np,P^*)$ $(c > 1)$, is determined to satisfy

$$(4.2) \qquad \inf_{\lambda'} \int_0^\infty F_{\lambda'}^{k-1} (cy) \, f_{\lambda'} (y) \, dy = P^*$$

where, now, $F_{\lambda'}(\cdot)$ and $f_{\lambda'}(\cdot)$ are the cdf and the density function of a non-central χ^2 with np degrees of freedom. Since the infimum of the above integral takes place when $\lambda' = 0$ (see Cor. 3.1.a in Gupta and Studden (1965)), the equation determining c is

$$(4.3) \qquad \int_0^\infty H_{p'}^{k-1}(cy)\, h_{p'}(y)\, dy = P^*, \quad p' = np$$

where $H_{p'}(x) = \int_0^x \frac{e^{-y}}{\Gamma(p')} y^{p'-1}\, dy$ and $\frac{d}{dx} H_{p'}(x) = h_{p'}(x)$.

The values of $c' = 1/c$ satisfying (4.3) are given by Gupta (1963) for selected values of p' and P^* (see Table 1, $p' = \nu/2$). Armitage and Krishnaiah (1964) have extensive tables for c'.

The rule for selecting the population with the minimum value of λ is defined by

select Π_i iff

$$\Sigma\, y_{ij} \leq b \min_i \left\{ \sum_{j=1}^{n} y_{ij};\ \ i = 1,2,\ldots, k \right\} .$$

The constant $b = b(k,np, p^*)$, in this case, is given by

$$(4.4) \qquad \int_0^\infty \left[1 - H_{p'}\left(\frac{y}{b}\right)\right]^{k-1} h_{p'}(y)\, dy = P^*, \quad p' = np \quad .$$

The values of $b' = 1/b$ satisfying (4.4) are tabulated in Gupta and Sobel (1962) for selected values of p' and P^* and more extensively by Krishnaiah and Armitage (1964).

<u>Selecting the Exponential Population with the Largest Mean Life or the Smallest Failure Rate</u>

Assume that the life model (time to failure) is the negative exponential with density $e^{-t/\theta}/\theta$. We are interested in the selection of the populations with large mean life $(=\theta)$ or small failure rate $1/\theta$ (the failure rate in this case is independent of time). Now, in many problems in reliability , the statistical inference is to be based on a truncated life test. Assume that the experimenter waits till a fixed number r of the total n units on life test fails so that for each of the k populations Π_i $(i = 1,2,\ldots, k)$, the ordered values $t_{i1} \leq t_{i2}, \leq, \ldots, \leq t_{ir}$ are available. A natural statistic in this case to consider is the total life statistic

$$T_i = t_{i1} + t_{i2} + \ldots + t_{ir} + (n-r)\, t_{ir} \tag{4.5}$$

It is known (see Epstein and Sobel (1953)) that this statistic is optimum for estimating θ. A selection rule based on T_i is as follows:

select population Π_i iff

$$c\, T_i \geq \max_{1 \leq j \leq k} T_j$$

Since $2T_i/\theta_i$ follows a gamma or χ^2 distribution with $2r$ degrees of freedom, the appropriate value of c can be found in Gupta (1963).

REFERENCES

1. Alam, K. and Rizvi, M.H. (1965). Selection from multivariate normal populations. Technical Report Number 65-1, Ohio State University.

2. Armitage, J.V. Major, and Krishnaiah, P.R. (1964). Tables for the Studentized largest chi-square distribution and their applications. ARL 64-188, Aerospace Research Laboratories, WP AFB, Ohio.

3. Bechhofer, R.E. (1954). A single-sample multiple decision procedure for ranking means of normal populations with known variances. Ann. Math. Statist. 25, 16-29.

4. Epstein, Benjamin and Sobel, M. (1953). Life Testing. J. Amer. Stat. Assn. 48, 486-502.

5. Gupta, S.S. (1963). On a selection and ranking procedure for gamma populations. Ann. Inst. Statist. Math. Tokyo 14, 199-216.

6. Gupta, S.S. and Sobel, M. (1962). On the smallest of several correlated F statistics. Biometrika 49, 509-523.

7. Gupta, S.S. (1965). On some selection and ranking procedures for multivariate normal populations using distance functions. Mimeograph Series No. 43, Department of Statistics, Purdue University. To appear in the Proceedings of the International Symposium on Multivariate Analysis, Academic Press.

8. Gupta, S.S. and Studden, W.J. (1965). On some selection and ranking procedures with applications to multivariate populations. Mimeograph Series No. 58, Department of Statistics, Purdue University. Accepted for publication in S.N. Roy Memorial Volume, Univ. of North Carolina Press.

9. Krishnaiah, P.R. and Armitage, J.V. (1964). Distribution of the studentized smallest chi-square with tables and applications. ARL 64-218, Aerospace Research Laboratories, WP AFB, Ohio.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Purdue University	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

3. REPORT TITLE
Some Aspects of Selection and Ranking Procedures with Applications

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Technical Report, July 1966

5. AUTHOR(S) *(Last name, first name, initial)*
Gupta, Shanti S. and Studden William J.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
July, 1966	13	9

8a. CONTRACT OR GRANT NO. NONR 1100(26)	9a. ORIGINATOR'S REPORT NUMBER(S) Mimeograph Series Number 81
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. AVAILABILITY/LIMITATION NOTICES
Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Also supported by Aerospace Research Laboratories Contract AF 33(657)11737	Logistics and Mathematical Science Branch Office of Naval Research Washington, D.C. 20360

13. ABSTRACT

In the density $f_{\theta_i}(x)$, let θ_i represent a physical characteristic which, in some sense, is a measure of the reliability of the ith population, $i=1,2,\ldots,k$. In many situations, the experimenter is interested in selecting a subset of the k populations which posses high reliability or large θ-values. In this type of selection, one wishes to ensure that the population with largest θ is contained in the selected subset (non-empty, small and of random size) with a given probability. Selection rules are suggested and studied when the parameter θ is: (i) a location parameter (ii) a scale parameter (iii) θ enters as a Poisson mixing parameter of the form $f_\theta(x) = \sum_{j=0}^{\infty} \frac{e^{-\theta}\theta^j}{j!} g_j(x)$ where $g_j(x)$ is a sequence of density functions. Properties of the selection rules are discussed. A result concerning the minimax character of (i) and (ii) is proved. Applications and examples are given.

DD FORM 1 JAN 64 1473

Unclassified
Security Classification

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Selection and Ranking Reliability, Failure Rate Translation and Scale Parameters Multivariate Normal Non-Central χ^2						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
_______________________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
_______________________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
_______________________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.