A Stochastic Model for Paired Comparisons of Social Stimuli

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S U M M A R Y

A stochastic model for paired comparisons of multi-attribute social stimuli is proposed where one objective is to find the relative importance of the attributes for a judge . It is related to the Bradley-Terry model where log-parameters are linear combinations of functions of the stimuli's attributes . This model does neither assume that the functions are fixed given in advance nor that different judges have the same set of functions . The choice among such functions however is admitted only within a finite given scope . Within the framework of exponential families maximum likelihood estimators and tests are derived and applied to data coming from a psychological experiment with schematic faces .

Key words : Paired comparisons ; Social Stimuli ; Exponential families ; Maximum likelihood estimators .

1. Introduction .

Almost all approaches of multi-attribute decision and judgment research consider the judment U as a function of the attributes $\mathcal{M}_k$ that characterize the objects G :

$$U (G^{(i)}) = F \left[a_1 f_1(\mathcal{M}_1^{(i)}) , \ldots , a_K f_K(\mathcal{M}_K^{(i)}) \right] , \qquad (1)$$

where $f_1 , \ldots , f_K$ are the functions which represent cognitively the K attributes of objects $G^{(1)} , \ldots , G^{(N)}$, and where $a_1 , \ldots , a_K$ are the weights which indicate the importance of the attributes $\mathcal{M}_1 , \ldots , \mathcal{M}_K$. The attributes jointly describe all objects by different aspects . F is a real valued monotone function in each of its arguments and specifies the composite effect of the attributes .

If the $\mathcal{M}_k$´s have scale values already , e.g. the test scores of intelligence , experience , or neuroticism of an applicant , if the f_k´s are the identity functions , and if the judgment U gets a value on the judgment criterion, e.g. the degree of the applicant´s qualification , weights a_k´s can be simply estimated from models , e.g. from the model of multiple linear regression (cf. Darlington (1968)) .

In many situations the decision between two objects $G^{(i)}$ and $G^{(j)}$ at a time which one of them is more attractive , more qualified , or rather more threatened , is psychologically more meaningful and less difficult for the judge than to assign a number on a rating scale . Bradley (1976) has given an excellent overview over the methods in the area of paired comparisons .

If the attributes $\mathcal{M}_1 , \ldots , \mathcal{M}_K$ have scale values while the judgments result from paired comparisons , Srinivasan and Shocker (1973) have proposed an estimation procedure for the weights $a_1 , \ldots , a_K$ from a system of linear inequalities .

However, if judgments are made with respect to paired comparisons of social objects, as to the attractiveness between two particular persons or holiday places, respectively, models for the estimation of importances $a_1, \dots, a_K$ should satisfy the following two conditions.

[1] It cannot be assumed that the attributes of the objects have already fixed functions $f_1, \dots, f_K$ being available for the judge.

Under this condition, the approach of Srinivasan and Shocker (1973) is not appropriate. Basically, the attributes could have got such functions from the judges, as it is assumed in several approaches. However, this procedure requires two conflicting judgment situations within the same judge:

(a) The decision between two objects (stimuli) $G^{(i)}$ and $G^{(j)}$ at a time, and

(b) The decision for scale values for each aspect of each respective attribute in the judgment criterion, e.g. in the attractiveness of a holiday place.

It is intuitively clear that these two judgment situations will mutually influence each other and thus will inflate the validity of the model.

[2] It cannot be assumed that different judges decide on the basis of the same set of functions $f_1, \dots, f_K$ for the attributes $\mathcal{M}_1, \dots, \mathcal{M}_K$ of the objects.

The aspects within the attributes often are preferred quite differently by different judges. For instance, one sea-side resort will be preferred by one judge due to its much frequented beaches while the other resort will be preferred by another judge due to its tranquil beaches. Averaging over judges would lead to results which represent no one of the judges. Thus an analysis of the judgments of the individual judge should be performed.

The objective of the present paper is to propose a stochastic model for the decision process as well as for the estimation of the weights a_k of the attributes $\mathcal{M}_k$, $k = 1, \ldots, K$. Its applicability will be demonstrated with paired comparisons of schematic faces in a psychological experiment . The final form of the model satisfies both conditions [1] and [2] .

In the second section normal equations for the maximum likelihood estimators will be derived as well as tests for hypotheses with respect to the a_k's . In the third section the results of the experiment will be analysed by the proposed procedures . The results corroborate the importance of condition [2] .

2. The Stochastic Model .

Let $G^{(1)}, \ldots, G^{(N)}$ be N stimuli (objects) which can be described by K attributes $\mathcal{M}_k \in \{1, 2, \ldots, m_k\}$ with fixed m_k , $k = 1, \ldots, K$. Thus they may be represented by

$$G^{(n)} = (\mathcal{M}_1^{(n)}, \ldots, \mathcal{M}_K^{(n)}) , \; n = 1, \ldots, N , \; N = \prod_{k=1}^{K} m_k . \qquad (2)$$

Each of the $N(N-1)/2$ pairs of stimuli are presented in a random succesion to a person (judge) who has to decide in each case which one of the two stimuli is preferred to the other one .

To fix ideas , let us assume at the beginning that the judge's decision process can be described by the following model , which can be viewed as being a specialization of the classical Bradley-Terry (1952) model . Here the $N(N-1)/2$ judgments are carried out mutually independently with probabilities $p_{ij}(\underline{a}) := \mathrm{pr}$ ($G^{(i)}$ is preferred to $G^{(j)}$) which depend on K unknown parameters $\underline{a} := (a_1, \ldots, a_K)$ in the following way :

$$p_{ij}(\underline{a}) = \exp\left\{\sum_{k=1}^{K} a_k \left[M_k^{(i)} - M_k^{(j)}\right]\right\} \Big/ \left(1 + \exp\left\{\sum_{k=1}^{K} a_k \left[M_k^{(i)} - M_k^{(j)}\right]\right\}\right) \quad (3)$$

$1 \le i < j \le N$, where $M_k^{(i)} = f_k (\mathcal{M}_k^{(i)})$, $i = 1, \ldots, N$, $k = 1, \ldots, K$, and $f_1, \ldots, f_K$ are fixed known real-valued functions .

Let the outcomes be random variables $X_{ij} \in \{0 , 1\}$, where $X_{ij} = 1$ (0) has the meaning that $G^{(i)}$ is preferred to $G^{(j)}$ ($G^{(j)}$ is preferred to $G^{(i)}$) , $1 \le i < j \le N$. Thus the likelihood function is given by

$$\mathrm{pr}\,(X_{ij} = x_{ij} \,,\; 1 \le i < j \le N)$$

$$= \frac{\exp\left\{\sum_{k=1}^{K} a_k \sum_{i<j} x_{ij} \left[M_k^{(i)} - M_k^{(j)}\right]\right\}}{\prod_{i<j}\left(1 + \exp\left\{\sum_{k=1}^{K} a_k \left[M_k^{(i)} - M_k^{(j)}\right]\right\}\right)} \,, \quad (4)$$

$$x_{ij} \in \{0 , 1\} \,,\; 1 \le i < j \le N \;.$$

It can be seen immediately that the following random vector $\underline{Z}$ is a sufficient statistic for $\underline{a} \in \mathbb{R}^K$:

$$\underline{Z} = (Z_1 , \ldots , Z_K) \,,$$

$$\text{where } Z_k = \sum_{i<j} X_{ij} \left[M_k^{(i)} - M_k^{(j)}\right] \,,\; k = 1, \ldots , K \;. \quad (5)$$

Moreover , the distributions of $\underline{Z}$ constitute a K - parametric exponential family , since for every fixed $\underline{a} \in \mathbb{R}^K$

$$\mathrm{pr}\,(\underline{Z} = \underline{z}) = \exp\left\{\sum_{k=1}^{K} a_k z_k + c(\underline{a}) + d(\underline{z})\right\} \,,$$

$$\underline{z} = (z_1, \ldots , z_K) \in \mathcal{Z} \,, \quad (6)$$

where $\mathcal{Z}$, $c(\underline{a})$ and $d(\underline{z})$ depend on $f_1 , \ldots , f_K$ in an obvious manner .

The parametrization is identifiable if and only if for every $k \in \{1, \ldots, K\}$ $m_k \geq 2$ and the values $f_k(1), \ldots, f_k(m_k)$ are not all identical . Under this condition an unique maximum likelihood estimator $\underline{\hat{a}} := (\hat{a}_1, \ldots, \hat{a}_K)$ for $\underline{a}$ exists if the following system of equations has a solution :

$$\sum_{i<j} \left[x_{ij} - p_{ij}(\underline{a}) \right] \left[M_k^{(i)} - M_k^{(j)} \right] = 0 \quad , k = 1, \ldots, K , \qquad (7)$$

which then is the unique solution of (7) and equals to $\underline{\hat{a}}$.

System (7) constitutes a system of normal equations which result from putting all partial derivations of the likelihood function (4) equal to zero and some additional standard analysis . That (7) in fact leads to maximum likelihood estimators can be seen more quickly by realizing that a solution $\underline{\hat{a}}(\underline{z})$ of (7) satisfies the canonical conditions for exponential families :

$$E_{\underline{\hat{a}}(\underline{z})}(Z_k) = z_k \quad , \quad k = 1, \ldots, K . \qquad (8)$$

Obviously , the form of a solution of (7) in case of her existence depends heavily on the form of the given $f_1, \ldots, f_K$. It cannot be given explicitly in general but has to be evaluated numerically in every concrete situation .

Remark 1 : If not all $N(N-1)/2$ pairs of stimuli but only a fixed subset of them are presented to the person and judged by her , similar results can be derived . The only thing that changes in the formulas is that all sums and products in (4) , (5) and (7) have to be restricted to this subset . However, the identifiability of the parametrization depends now additionally on this subset .

Remark 2 : If the $N(N-1)/2$ judgments are collected independently from R persons (judges) who can be assumed to judge according to (3) with a common $\underline{a}$, our results derived so far hold analogously . One only has to replace the

X_{ij}´s and x_{ij}´s throughout by the arithmetic means of the corresponding responses of the R persons and to pay attention to the fact that then (4) is no longer the likelihood function but now her R-th root . Especially , the model can be examined with generalized (maximum) likelihood ratio tests in two directions which can be established by use of the well known χ^2 - approximation . The first tests the hypothesis <u>H</u> : " $a_1 = \ldots = a_K = 0$ " (pure randomness) versus <u>K</u> : " model (3) " , and significance supports our model . The second tests the hypothesis <u>H</u>´: " model (3) " versus <u>K</u>´: " Bradley-Terry model (1952) " , and non-significance then supports our model.Of course, one should clearly distinguish between the meaning of " supports " in the two tests due to the well known unsymmetry in the theory of testing hypotheses . It should be pointed out also that in case of R = 1 person both tests are not at hand and even the parameters in the Bradley-Terry model (1952) are then mostly not estimable within the maximum likelihood approach .

Instead of proceeding along the lines as indicated in Remark 2 , we prefer to generalize our model , admitting a more individual behavior of the single persons . By this we do not only mean that parameters $\underline{a}$ may now differ from person to person but also that,instead of having $\underline{f} := (f_1 , \ldots , f_K)$ fixed known , we assume now that only $f_k \in \mathfrak{F}_k$ holds , where $\mathfrak{F}_k$ is a given finite set of real-valued functions , $k = 1, \ldots , K$. Maximum likelihood estimates $\hat{\underline{f}} := (\hat{f}_1 , \ldots , \hat{f}_K)$ and $\hat{\underline{a}}$ for a single person are determined now in the following way : For every fixed $\underline{f}$ with $f_k \in \mathfrak{F}_k$, $k = 1, \ldots , K$ a solution of (7) has to be found and , together with $\underline{f}$, to be inserted into the likelihood function (4) . The largest value of (4) then determines the maximum likelihood estimates $\hat{\underline{f}}$ and $\hat{\underline{a}}$.

As long as the sizes of the sets $\mathfrak{F}_1 , \ldots , \mathfrak{F}_K$ are not too large , the following single tests for every fixed $\underline{f}$ can be combined to a simultaneous test for

$\underline{H}$: " $a_1 = \ldots = a_K = 0$ " . Its p - value can be bounded from above in the usual manner with the help of the single tests´ p - values and Bonferroni´s inequality . Thus , let $\underline{f}$ be fixed for a moment and let us look for a test for $\underline{H}$ versus the general alternative that $\underline{H}$ is not true . For every $k \in \{1, \ldots, K\}$ for the testing problem $\underline{H_k}$: " $a_k = 0$ " versus $\underline{K_k}$: " $a_k \neq 0$ " there exists an UMPU - test who rejects $\underline{H_k}$ if Z_k falls outside of an interval . But the boundaries of the interval depend not only on the level but also on the values of $Z_1, \ldots, Z_{k-1}, Z_{k+1}, \ldots, Z_K$ (cf. Lehmann (1959) p. 134) . Thus in view of the combinatorial difficulties arising from $\underline{f}$, these tests are practically not performable in general .

Alternatively , let us propose the following asymptotic tests for $\underline{H}$ (instead of $\underline{H_k}$) versus $\underline{K_k}$, k = 1, ... , K . The single tests hereby are based on statistics

$$\sum_{i<j} \left[X_{ij} - 1/2\right]\left[M_k^{(i)} - M_k^{(j)}\right] \Big/ \left(1/4 \sum_{i<j}\left[M_k^{(i)} - M_k^{(j)}\right]^2 \right)^{1/2} \qquad (9)$$

and acceptance regions $\left[\Phi^{-1}(\alpha/2K),\ \Phi^{-1}(1 - \alpha/2K)\right]$, k = 1, ... ,K , where Φ denotes the standard normal cumulative distribution function and α the level of the corresponding test . A necessary and sufficient condition for the asymptotic normality of the test statistics in a sequence of models (2) and (3) (triangular array) is that for large N the following terms tend to 0 :

$$\max_{r<s} \left[M_k^{(r)} - M_k^{(s)}\right]^2 \Big/ \sum_{i<j} \left[M_k^{(i)} - M_k^{(j)}\right]^2 , \quad k = 1, \ldots , K . \qquad (10)$$

This since under $\underline{H}$ the X_{ij} , $1 \leq i < j \leq N$, are independently identically distributed Bernoulli - variables with parameter 1/2 , and therefore the conditions are equivalent to the Lindeberg condition (cf. Feller (1971) p.264 (f)) .

Remark 3 : To reduce the loss of power induced by the use of Bonferroni´s inequality other testing procedures are thinkable , e.g. those which are based on the joint normality of the K statistics given in (9) .

3. Applications to a Psychological Experiment .

The objects (stimuli) of the psychological experiment are schematic faces with K = 3 attributes : " Mouth " $\mathcal{M}_1 \in \{1\ (\frown)\ ,\ 2\ (-)\ ,\ 3\ (\smile)\}$, " Hair " $\mathcal{M}_2 \in \{1$ (thin) , 2 (short) , 3 (full) $\}$, and " Eyes " $\mathcal{M}_3 \in \{1\ (-)\ ,\ 2\ (\bullet)\}$, (see Figure 1) .

Please insert Figure 1 here

Each pair of the N = 18 faces has been presented to the person (judge) by projecting the two corresponding slides onto a sreen in a random ordering . The person decided spontaneously which one of the two faces looked more likable by crossing against one of two little boxes . In tnis manner $N(N-1)/2 = 153$ decisions of the person have been recorded . 36 persons have participated in groups of 4 in this experiment .

To get the numerical analysis manageable we restrict ourselves to functions which lead to equidistant scales . Under this restriction the choice of the admissible functions f_1 , f_2 , f_3 can be restricted to the following classes without further loss of generality :

$$\begin{aligned} \mathcal{F}_1 = \{p, q, r\} \quad \text{where} \quad & p(1) = 1,\ p(2) = 2,\ p(3) = 3 \\ & q(1) = 2,\ q(2) = 1,\ q(3) = 3 \\ \text{and} \quad & r(1) = 1,\ r(2) = 3,\ r(3) = 2, \end{aligned} \qquad (11)$$

$$\mathcal{F}_2 = \mathcal{F}_1 \quad \text{and} \quad \mathcal{F}_3 = \{s\} \quad \text{where} \quad s(1) = 1,\ s(2) = 2 .$$

For each fixed $k \in \{1, 2, 3\}$ the justification is as follows : In the model (2) , (3) the $M_k^{(i)}$ - values appear only in terms of differences .

Thus their smallest value can be set equal to 1 . A γ- fold of these differences can be compensated in the parameter a_k by a change to a_k/γ . Thus their largest value can be set equal to 3 . Finally , a complete reverse of the three values (e.g. (1 , 3 , 2) to (3 , 1 , 2)) can be compensated by a change of the sign of a_k (for more details see also 2) below) . The results are given in Table 1 .

Please insert Table 1 here

---

1) The estimated functions $\hat{f}_1$ and $\hat{f}_2$ obviously confirm our condition [2] . A comparison of the $\hat{a}$'s among persons having the same functions $\hat{f}_1$ and $\hat{f}_2$ and signs in the $\hat{a}_k$'s (cf. 2)) demonstrates additional individual behavior .

2) A negative sign of an a_k means that the person shows the opposite preference $\tilde{f}_k$ ($\tilde{f}_k = h \circ f_k$ with $h(1) = 3$, $h(2) = 2$, $h(3) = 1$) to f_k within the attribute $\mathcal{M}_k$, $k = 1 , 2 , 3$. The absolute values of the parameters a_1 , a_2 , a_3 of a person indicate which relative importance the three attributes exert on the decision process . A negative sign of an $\hat{a}_k$ and the absolute values of the $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$ are thus to be interpreted analogously .

Remark 4 : For person No. 7 maximum likelihood estimates are not available . The data reflect a deterministic behavior with respect to $\mathcal{M}_1$ and neglect of $\mathcal{M}_2$ and $\mathcal{M}_3$ as long as the two faces in a pair differ with respect to $\mathcal{M}_1$. For person No. 17 both q and r are maximum likelihood estimates for f_2 , and the sign of $\hat{a}_2$ depends on the choice among them . For person No. 19 both p and q are maximum likelihood estimates for f_2 .

3) Column " Subj. " gives the subjective rank order of the importance of the attributes for each person . They have been requested in interviews following the experiment . For instance , (1 , 3 , 2) is to be understood that the person rated $\mathcal{M}_2$ as most and $\mathcal{M}_1$ as least important for her judgements . If a person rated all attributes as equally important this is denoted by a (- , - , -).

Remark 5 . As a measure of the difference between the subjective rank order of a person and her estimated rank order (cf. 2)) let us take the number of inversions . Among those persons who could give a subjective rank order a χ^2 - test with 3 degrees of freedom can be performed for the hypothesis of " Randomness " versus the alternative that less inversions are more likely . The p - value for these 30 persons (person No. 7 is treated conservatively) is p = 0.0268 .

4) The last column gives the upper bounds for the p - values of the proposed test (" Randomness " versus " Model ") for every person . Each value by Bonferroni´s inequality is calculated as the 7 - fold of the minimum of the p - values of the single tests which are taken from Owen (1962) .

Remark 6 . The numerical calculations have been performed on the H B 66/80 of the University of Mainz . The solutions of the system of equations (7) are based on a procedure of Werner (1979) .

A C K N O W L E D G E M E N T

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Figure 1

The 18 schematic faces of the reported psychological experiment

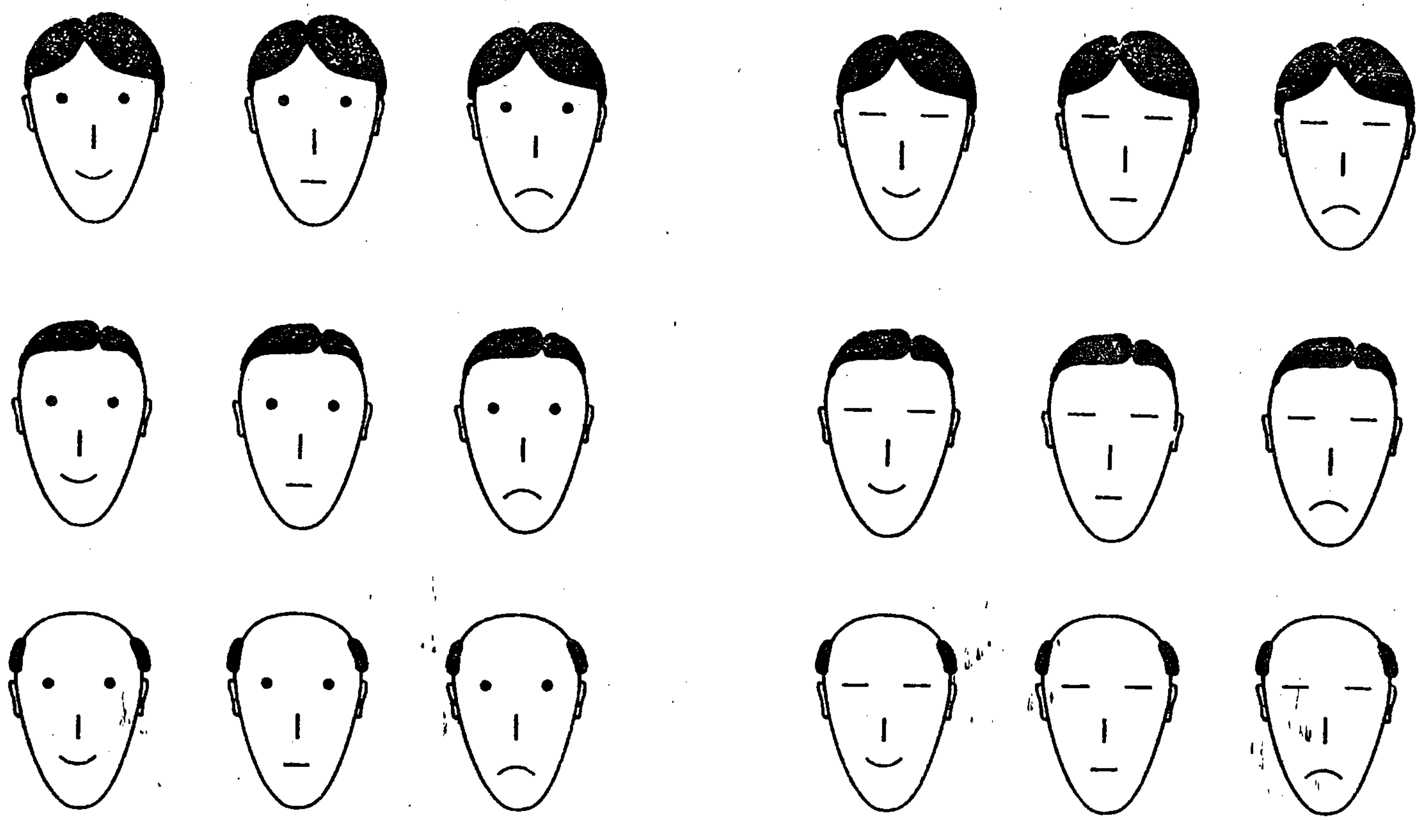

Table 1

Estimates, subjective rank orders, and p-values of 36 persons

Person	$(\hat{f}_1, \hat{f}_2)$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	Subj.	Test
1	(p,p)	8.7230	2.7481	1.2790	(3,1,2)	$5.60 \cdot 10^{-21}$
2	(p,p)	5.5402	1.8673	1.7035	(2,1,3)	$1.59 \cdot 10^{-18}$
3	(p,q)	4.2700	1.5842	1.5627	(1,3,2)	$5.59 \cdot 10^{-17}$
4	(p,q)	2.8581	1.1590	1.6735	(3,1,2)	$9.54 \cdot 10^{-13}$
5	(p,p)	3.2006	1.1475	1.0484	(3,1,2)	$3.13 \cdot 10^{-16}$
6	(q,p)	1.8752	1.0757	0.6223	(3,2,1)	$4.22 \cdot 10^{-12}$
7	(p,-)	∞	--	--	(3,1,2)	$5.60 \cdot 10^{-21}$
8	(q,p)	2.2223	0.7462	1.5745	(2,1,3)	$7.33 \cdot 10^{-11}$
9	(p,q)	4.6310	1.8901	0.4377	(3,2,1)	$2.51 \cdot 10^{-19}$
10	(p,p)	2.4133	0.7431	2.0370	(1,3,2)	$2.88 \cdot 10^{-10}$
11	(p,p)	6.3412	2.4086	0.4456	(-,-,-)	$3.82 \cdot 10^{-20}$
12	(p,p)	2.4624	2.5545	0.7648	(2,1,3)	$2.88 \cdot 10^{-10}$
13	(p,r)	1.6597	-1.9288	1.0031	(-,-,-)	$3.96 \cdot 10^{-9}$
14	(p,q)	3.8592	0.9703	1.6856	(2,1,3)	$3.13 \cdot 10^{-16}$
15	(p,q)	2.9308	-0.2548	1.4654	(3,1,2)	$8.71 \cdot 10^{-15}$
16	(q,r)	0.7244	-0.3808	1.0754	(2,1,3)	$1.09 \cdot 10^{-9}$
17	(p,q or r)	1.7691	±1.6961	0.8527	(-,-,-)	$2.55 \cdot 10^{-8}$
18	(q,r)	1.4558	-1.0516	1.3633	(1,3,2)	$1.39 \cdot 10^{-8}$
19	(p,p or q)	1.1351	0.9076	0.1316	(2,1,3)	$8.39 \cdot 10^{-8}$
20	(p,r)	1.7985	-0.6672	1.2310	(-,-,-)	$1.09 \cdot 10^{-9}$
21	(p,p)	1.8366	1.8763	1.0929	(3,2,1)	$4.65 \cdot 10^{-8}$
22	(p,p)	2.6216	2.3006	0.8293	(1,3,2)	$7.33 \cdot 10^{-11}$
23	(p,p)	2.9433	1.9670	0.6170	(2,3,1)	$4.34 \cdot 10^{-14}$
24	(p,p)	1.6562	-0.4739	0.2743	(3,1,2)	$2.08 \cdot 10^{-13}$
25	(p,p)	2.3590	0.6951	0.5175	(3,1,2)	$3.13 \cdot 10^{-16}$
26	(p,q)	2.1148	1.8883	0.3272	(2,3,1)	$7.33 \cdot 10^{-11}$
27	(q,q)	0.8842	1.1139	0.3190	(1,3,2)	$2.66 \cdot 10^{-7}$
28	(p,p)	1.2671	0.2191	0.3805	(3,1,2)	$2.88 \cdot 10^{-10}$
29	(p,p)	2.6836	-0.1918	1.3614	(2,1,3)	$8.71 \cdot 10^{-15}$
30	(p,p)	2.5130	0.3020	0.7664	(3,1,2)	$3.13 \cdot 10^{-16}$
31	(p,q)	0.6130	1.8514	2.5592	(1,2,3)	$1.69 \cdot 10^{-15}$
32	(p,r)	1.3106	-1.2606	2.4690	(1,2,3)	$1.69 \cdot 10^{-15}$
33	(p,q)	0.1412	0.4580	-0.2010	(-,-,-)	$1.36 \cdot 10^{-2}$
34	(r,r)	-0.8450	-1.4463	-0.1024	(-,-,-)	$7.33 \cdot 10^{-11}$
35	(p,r)	3.0601	0.4474	2.6794	(2,1,3)	$4.22 \cdot 10^{-12}$
36	(p,p)	1.6102	0.3132	0.4356	(3,1,2)	$9.54 \cdot 10^{-13}$

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number): Paired comparisons; Social Stimuli; Exponential families; Maximum likelihood estimators.
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A stochastic model for paired comparisons of multi-attribute social stimuli is proposed where one objective is to find the relative importance of the attributes for a judge. It is related to the Bradley-Terry model where log-parameters are linear combinations of functions of the stimuli's attributes. This model does neither assume that the functions are fixed given in advance nor that different judges have the same set of functions. The choice among such functions however is admitted only within a finite given scope. Within the framework of exponential families maximum likelihood estimators and tests are derived and applied to data

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) coming from a psychological experiment with schematic faces.