ON ISOTONIC SELECTION RULES FOR BINOMIAL POPULATIONS BETTER THAN A STANDARD*

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ON ISOTONIC SELECTION RULES FOR BINOMIAL POPULATIONS BETTER THAN A STANDARD*

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ABSTRACT

The problem of selecting a subset containing all binomial populations better than a standard is considered. It is assumed that the success probabilities associated with the experimental populations follow a simple prior ordering; however, the actual values of these parameters are unknown. Selection of any subset which includes all better populations is called a correct selection. We restrict our attention to isotonic procedures that guarantee a specified minimum probability of a correct selection. Both cases of the standard parameter known and unknown have been considered. When the standard parameter is unknown, the proposed procedure is conservative. Some tables of constants associated with the proposed isotonic procedures are given.

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0. INTRODUCTION

The problem of selecting populations better than a control with respect to a location parameter under ordering prior has been considered by Gupta and Yang (1981). In this paper we consider the case of binomial populations (important for discrete data) for which the parameters of interest are not the location parameters as was the case studied by Gupta and Yang (1981). We consider both cases when the parameter of control is known and unknown. For the case of known control, we propose an isotonic procedure which is given in Section 2.1. The results in this section deal with two cases, namely, the sample sizes all equal, and unequal. Some recursive relations are derived for computing the constants required for the proposed procedure. When the control is unknown, a conditional isotonic procedure is proposed in Section 2.2. This procedure provides a conservative solution for the unconditional procedure. Brief tables of associated constants for the proposed procedures are given in Table I and Table II.

1. NOTATIONS AND DEFINITIONS

Assume $\pi_0, \pi_1, \ldots, \pi_k$ are all binomial populations such that π_i has density $b(m; p_i)$, $i = 0,1,2,\ldots,k$. It is assumed that $p_1 \leq p_2 \leq \cdots \leq p_k$, however, the actual values of these p_i's are not known. We consider π_0 as a control and our goal is to select a subset of these k populations so that all "good" populations are included in the subset selected, where π_i is considered "good" if and only if $p_i \geq p_0$. By a correct selection (CS) we mean the selection of any non-trivial subset which contains all good populations.

Let $\Omega = \{(p_0,p_1,\ldots,p_k) | 0 < p_1 \leq p_2 \leq \cdots \leq p_k < 1,\ 0 < p_0 < 1\}$. Let us denote the sets $a_i = \{i, i+1,\ldots,k\}$, $1 \leq i \leq k$ and $a_0 \equiv \phi$ (the empty set). If a_i is taken, it means the subset $\{\pi_i, \pi_{i+1}, \ldots, \pi_k\}$ of the k populations is selected. Since by our assumption that p_i are ordered according to an ascending (simple) ordering prior, it is therefore appropriate to restrict ourselves to the action space $\mathcal{A} = \{a_0, a_1, a_2, \ldots, a_k\}$. For given positive integer $n_0, n_1, \ldots, n_k$, we assume n_i independent observations are drawn from π_i $(i = 0,1,2,\ldots,k)$ which are, respectively, denoted by $X_{i1}, X_{i2}, \ldots, X_{in_i}$. The sample space is denoted by

$$\mathcal{X} = \{\underset{\sim}{X} \in I^{n_0+n_1+\ldots+n_k} | \underset{\sim}{X} = (X_{01}, X_{02}, \ldots, X_{0n_0}, X_{11}, \ldots, X_{1n_1}, \ldots, X_{k1}, \ldots, X_{kn_k}\}$$

where I denote the set of non-negative integers.

We restrict ourselves to isotonic selection procedures δ which satisfy the P*-condition, i.e. $\inf_{\underset{\sim}{\theta} \in \Omega} P_{\underset{\sim}{\theta}}(CS|\delta) \geq P^*$ where P* is a preassigned value.

A poset $(S, \underset{\sim}{<})$ denotes a non-empty set S with a binary partial order $\underset{\sim}{<}$ defined on it. A real-valued function f defined on a poset $(S, \underset{\sim}{<})$ is called isotonic if f preserves the order on S, i.e. $x \leq y$ implies $f(x) \leq f(y)$. Let g be a real-valued function and let W be a positive-valued function, both defined on a poset $(S, \underset{\sim}{<})$. An isotonic function g* on S is called an isotonic regression of g with weight W if $\sum_{x \in S} [g(x)-g^*(x)]^2 W(x)$ attains its minimum values over set of all isotonic functions on S. It is well-known (see Barlow, Bartholomew, Bremner and Brunk (1972)) that there exists one and only one isotonic regression of a given g with a given weight W defined on S. Some algorithms such as "pool-adjacent-violators" or the so-called "up-and-down blocks" are referred to in Barlow et al (1972) and Ayer et al (1955).

Let $n_0 = 0$ and $n_1 = n_2 = \ldots = n_k = n$ and $m = 1$. Let $\bar{X}_i$ denote the sample mean from π_i, $i = 1,2,\ldots,k$. The isotonic regression of $\bar{X}_i$ with common weight n is a maximum likelihood estimate for $\underset{\sim}{p} = (p_1, p_2, \ldots, p_k)$ which is given by the following theorem.

Theorem 1.1 (see Barlow et al (1972), p. 102)

The maximum likelihood estimate for $\underset{\sim}{p} = (p_1, p_2, \ldots, p_k)$ with $p_1 \leq p_2 \leq \cdots \leq p_k$ is given by the isotonic regression of $\bar{X}_i$ with common weight n, i.e. $\hat{\underset{\sim}{X}} = (\hat{X}_{1,k}, \hat{X}_{2,k}, \ldots, \hat{X}_{k,k})$ minimizes $\sum_{i=1}^{k} (\bar{X}_i - p_i)^2 n$ where, by the max-min formula of Ayer et al (1955), we have

$$(1.1) \qquad \hat{X}_{i,k} = \max_{1 \leq s \leq i} \{ \min_{s \leq t \leq k} ((\bar{X}_s + \bar{X}_{s+1} + \ldots + \bar{X}_t)/(t-s+1))\}$$

$$= \max_{1 \leq s \leq i} \hat{\hat{X}}_{s,k}$$

where

$$(1.2) \qquad \hat{\hat{X}}_{s,k} = \min\{\bar{X}_s, \frac{1}{2}(\bar{X}_s + \bar{X}_{s+1}), \ldots, (\bar{X}_s + \bar{X}_{s+1} + \ldots + \bar{X}_k)/(k-s+1)\}.$$

2. ISOTONIC SELECTION PROCEDURES

2.1 Isotonic Selection Procedure δ_1 When p_0 Is Known

a. Equal Sample Size Case

Since p_0 is known, we take $n_0 = 0$; assume $n_1 = n_2 = \ldots = n_k = n$. Without loss of generality, we may consider $m = 1$, i.e. π_i is a Bernoulli (p_i). For given positive constants $d_1, d_2, \ldots, d_k$ $(0 < d_i < p_0$, to be determined later), we propose the following unconditional procedure δ_1: $\delta_1(\underset{\sim}{X}) = a_{\epsilon(\underset{\sim}{X})}$, where $\epsilon(\underset{\sim}{X}) = \min\{i \mid \hat{X}_{i,k} \geq p_0 - d_i\}$ and $\hat{X}_{i,k}$ is defined by (1.1). Since $\delta_1(\underset{\sim}{X})$ depends on vector $\underset{\sim}{d} = (d_1, d_2, \ldots, d_k)$, we may denote it by $\delta_1(\underset{\sim}{d})$ when there is no confusion.

Determination of $\underset{\sim}{d}$ for $\delta_1(\underset{\sim}{d})$. In order to satisfy the basic P*-condition, we need to compute $\inf_{\Omega} P(CS \mid \delta_1(\underset{\sim}{d}))$. For notational conveniences, we define

$$\Omega_i = \{\underset{\sim}{p} \in \Omega \mid p_{k-i} < p_0 \leq p_{k-i+1}\}, \quad i = 1, 2, \ldots, k-1$$

$$\Omega_k = \{\underset{\sim}{p} \in \Omega \mid p_0 \leq p_1\}$$

and

$$\Omega_0 = \{\underset{\sim}{p} \in \Omega | p_k < p_0\}.$$

Then, Ω_i are disjoint and $\Omega = \bigcup_{i=0}^{k} \Omega_i$. Again for notational convenience, when there is no confusion, we denote, respectively, $\hat{X}_{i,k}$ and $\hat{\hat{X}}_{j,k}$ by $\hat{X}_i$ and $\hat{\hat{X}}_j$ for a given fixed k. Then, for any $\underset{\sim}{p} \in \Omega_i$,

$$\text{(2.1)} \qquad P_{\underset{\sim}{p}}(CS|\delta_1(\underset{\sim}{d})) = P_{\underset{\sim}{p}}\{\bigcup_{j=1}^{k-i+1} \{\hat{X}_j \geq p_0 - d_j\}\}$$

$$= P_{\underset{\sim}{p}}\{\bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^{j} (\hat{\hat{X}}_r \geq p_0 - d_j)\}$$

$$\geq P_{\underset{\sim}{p}}\{\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1}\}.$$

Since $P_{\underset{\sim}{p}}(\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1})$ is increasing in p_{k-i+j} for $j = 1,2,\ldots,i$, keeping all other (i-1) parameters fixed, hence, the right hand side of (2.1) attains its minimum at $p_{k-i+1} = p_{k-i+2} = \ldots = p_k = p_0$. On the other hand, if we take a special vector

$$\text{(2.2)} \qquad \underset{\sim}{p}_0 = (p_0, \overbrace{0,0,\ldots,0}^{k-i}, p_0, p_0, \ldots, p_0) \in \bar{\Omega}_i, \text{ the closure of } \Omega_i,$$

then, we see that

$$P_{\underset{\sim}{p}_0}(\bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^{j} (\hat{\hat{X}}_r \geq p_0 - d_j)) = P_{\underset{\sim}{p}_0}(\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1})$$

since $\bar{X}_j = 0$ a.s. when $p_j = 0$ for $j = 1,2,\ldots,k-i$. Hence, $\inf_{\Omega_i} P(CS|\delta_1(\underset{\sim}{d}))$ attains at $\underset{\sim}{p} = \underset{\sim}{p}_0$ as defined by (2.2). Again,

$$\text{(2.3)} \qquad \inf_{\Omega} P_{\underset{\sim}{p}}(CS|\delta_1(\underset{\sim}{d})) = \min_{1 \leq i \leq k} \{\inf_{\Omega_i} P_{\underset{\sim}{p}}(CS|\delta_1(\underset{\sim}{d}))\}$$

because for $\underset{\sim}{p} \in \Omega_0$, any action in $\mathcal{G}$ is, according to our definitions, a correct selection. Therefore, if for each i ($1 \leq i \leq k$) $\inf_{\Omega_i} P_{\underset{\sim}{p}}(CS|\delta_1(\underset{\sim}{d}) \geq P^*$, then the

P*-condition holds for $\delta_1(\underset{\sim}{d})$. Now,

(2.4) $$\inf_{\Omega_i} P\{CS|\delta_1\} = P_{\underset{\sim}{p}_0}\{\hat{\hat{X}}_{k-i+1} \geq p_0 - d_{k-i+1}\}$$

$$= P_{\underset{\sim}{q}_0}(\hat{\hat{X}}_{1,i} \geq p_0 - \alpha_i)$$

where

$$\underset{\sim}{q}_0 = \{p_0, p_0, \ldots, p_0\} \in (0,1)^i, \text{ and } \alpha_i = d_{k-i+1}.$$

It follows from (1.2) and (2.4) that

(2.5) $$\inf_{\Omega_i} P\{CS|\delta_1(\underset{\sim}{d})\} = P_r\{Y_1 \geq c_i, \tfrac{1}{2}(Y_1+Y_2) \geq c_i, \ldots, \tfrac{1}{i}(Y_1+Y_2+\ldots+Y_i) \geq c_i\}$$

where Y_1, $Y_2, \ldots, Y_i$ are i.i.d. with Y_1 being $b(n; p_0)$ and $c_i = n(p_0 - \alpha_i)$. Hence, it suffices to compute

(2.6) $$a_i(\alpha_i) \equiv P_r\{Y_1 \geq e_1, Y_1 + Y_2 \geq e_2, \ldots, \sum_{j=1}^{i} Y_j \geq e_i\}$$

where

(2.7) $$e_j = j\, c_i = jn(p_0 - \alpha_i), \qquad j = 1,2,\ldots,i.$$

Define

(2.8) $$a_j(\alpha_i) = P_r\{Y_1 \geq e_1, Y_1 + Y_2 \geq e_2, \ldots, \sum_{r=1}^{j} Y_r \geq e_j\}, \ j = 1,2,\ldots,i.$$

Letting $<\alpha> = -[-\alpha]$, i.e. the smallest integer no less than α, we have the following useful lemma.

<u>Lemma 2.1</u>. (i) $a_1(\alpha_1) = \sum_{r=r_0}^{n} \binom{n}{r} p_0^r (1-p_0)^{n-r}$ where

$$r_0 = <n(p_0 - \alpha_1)>$$

$$\text{(ii)}\quad a_j(\alpha_i) = \sum_{r_1=\langle c\rangle}^{n} g(r_1)\{\sum_{r_2=\langle 2c-r_1\rangle}^{n} g(r_2)\{\sum_{r_3=\langle 3c-r_1-r_2\rangle}^{n} g(r_3) \cdots$$

$$\{\sum_{r_j=\langle jc-r_1-r_2\cdots r_{j-1}\rangle}^{n} g(r_j)\}\}\},\quad j = 1,2,\ldots,i$$

where

$$c = n(p_0-\alpha_i) \text{ and}$$

$$g(r) = \binom{n}{r}p_0^r(1-p_0)^{n-r}.$$

Proof: To compute (i) is straightforward. To prove (ii), define

$$A_j(\alpha,\beta) = \Pr\{Y_1 \geq \alpha,\ Y_1 + Y_2 \geq \alpha+\beta,\ldots,\sum_{r=1}^{j} Y_r \geq \alpha + (j-1)\beta\}$$

where $Y_1,Y_2,\ldots,Y_j$ are iid $b(n;\ p_0)$. Conditioning on $Y_1 = r_1$, we obtain

$$A_j(\alpha,\beta) = \sum_{r=\langle\alpha\rangle}^{n} \binom{n}{r}p_0^r(1-p_0)^{n-r}\Pr\{Y_2 \geq \alpha+\beta-r_1,\ Y_2+Y_3 \geq \alpha+2\beta-r_1,$$

$$\ldots,\ \sum_{r=2}^{j} Y_r \geq \alpha + (j-1)\beta-r_1\}$$

$$= \sum_{r_1=\langle\alpha\rangle}^{n} \binom{n}{r}p_0^r(1-p_0)^{n-r}A_{j-1}(\alpha+\beta-r_1,\beta).$$

Taking $\alpha = \beta = n(p_0-d)$ and using (i) and by mathematical induction, we obtain (ii).

Computations of α_i in Table I. Lemma 2.1 gives a direct method of computing $a_i(\alpha_i)$. For given n, i, p_0 and P^*, we start with $v_1 = p_0 - \frac{1}{in}$ and compute $a_i(v_1)$. If $a_i(v_1) < P^*$, we take $\alpha_i = v_1 + \frac{1}{in} = p_0$; otherwise, we take $v_2 = v_1 - \frac{1}{in} = p_0 - \frac{2}{in}$, and compute $a_i(v_2)$. If $a_i(v_2) < P^*$, we take $\alpha_i = v_2 + \frac{1}{in} = p_0 - \frac{1}{in}$; otherwise take $v_3 = v_2 - \frac{1}{in}$. This process continues until for the first time $a_i(v_{r-1}) \geq P^*$ and $a_i(v_r) < P^*$. Then, we stop and take $\alpha_i = v_{r-1} = p_0 - \frac{r-1}{in}$. The reason that each time we decrease by $\frac{1}{in}$ is that (2.5) remains unchanged as long as the value of ic_i changes by an amount less than 1, i.e. $in(p_0-\alpha_i) < 1$. The smallest values of α_i satisfying $a_i(\alpha_i) \geq P^*$ are tabulated in Table I for $n = 5(1)10$, $p_0 = 0.1(0.1)0.5$, $P^* = 0.90, .95$, and $i = 1(1)4$. For $p_0 = 0.6(0.1)0.9$, the problem can be treated by considering failures instead of successes. It should be noted that $d_{k-i+1} = \alpha_i$, $i = 1,\ldots,k$.

Now define,

$$a'_j(\alpha,\beta) = P\{n(p_0-\alpha) \leq Y_1 \leq n(p_0-\beta),\ 2n(p_0-\alpha) \leq Y_1+Y_2 \leq 2n(p_0-\beta),$$

$$\ldots, jn(p_0-\alpha) \leq \sum_1^j Y_r \leq jn(p_0-\beta)\}.$$

Then, from analogous arguments as in Lemma 2.1, we have

<u>Corollary 2.1</u>: $$a'_j(\alpha,\beta) = \sum_{r_1=\langle c_1\rangle}^{[c_2]} g(r_1)\{\sum_{r_2=\langle 2c_1-r_1\rangle}^{[2c_2-r_1]} g(r_2)\{\sum_{r_3=\langle 3c_1-r_1-r_2\rangle}^{[3c_2-r_1-r_2]} g(r_3)$$

$$\ldots\{\sum_{r_j=r_{0j}}^{r'_{0j}} g(r_j)\}\}\},$$

where

$$c_1 = n(p_0-\alpha)$$

$$c_2 = n(p_0-\beta)$$

$$r_{0j} = \langle jc_1-r_1-r_2-\ldots-r_{j-1}\rangle, \quad j = 2,3,\ldots$$

$$r'_{0j} = [jc_2-r_1-r_2-\ldots-r_{j-1}].$$

Computations of α and β such that $a'_i(\alpha,\beta) \geq P^*$ are analogous to those of d_i for Table I.

b. Unequal Sample Sizes Case

We also take $n_0 = 0$ since p_0 is known, and assume $m = 1$. Then the isotonic estimates in (1.1) and (1.2) become

$$\hat{X}_{i,k} = \max_{1\le s\le i} \hat{\hat{X}}_{s,k}$$

where

$$\hat{\hat{X}}_{s,k} = \min\{\bar{X}_s,\ (n_s\bar{X}_s + n_{s+1}\bar{X}_{s+1})/(n_s+n_{s+1}),\ldots,\ (n_s\bar{X}_s+n_{s+1}\bar{X}_{s+1}+\ldots+n_k\bar{X}_k)/(n_s+n_{s+1}+\ldots+n_k)\}. \tag{2.9}$$

For our notational simplicity, we define

$$\underline{i} = k-i+1 \text{ and}$$

$$m_{i,j} = n_i+n_{i+1} +\ldots+ n_{i+j-1}. \tag{2.10}$$

Then (2.5) becomes

$$\inf_{\Omega_i} P\{CS|\delta_1\} = P\{Z_1 \ge c_1,\ Z_1+Z_2 \ge c_2,\ldots,Z_1+Z_2+\ldots+Z_i \ge c_i\} \tag{2.11}$$

where

$$\begin{aligned} c_1 &= m_{\underline{i},1}(p_0-\alpha_i)\\ c_2 &= m_{\underline{i},2}(p_0-\alpha_i)\\ &\ldots\ldots\ldots\ldots\ldots\ldots\\ c_i &= m_{\underline{i},i}(p_0-\alpha_i) \end{aligned} \tag{2.12}$$

and $Z_1,Z_2,\ldots,Z_i$ are independent with Z_j being $b(n_{k-i+j};\ p_0)$. Then, (2.11) can be computed according to the following theorem. Define

$$b_{j,i}(\underset{\sim}{c}) = P\{Z_1 \ge c_1,\ Z_1+Z_2 \ge c_2,\ldots,\sum_{r=1}^{j} Z_r \ge c_j\},\quad j = 1,2,\ldots,i, \tag{2.13}$$

where $\underset{\sim}{c} = (c_1,c_2,\ldots,c_i)$ and Z_i's are defined by (2.12). Then, we have

Theorem 2.1. $b_{j,i}(\underset{\sim}{c}) = \sum_{r_1=<c_1>}^{n_{k-i+1}} g(n_{k-i+1},r_1)\{\sum_{r_2=<c_2-r_1>}^{n_{k-i+2}} g(n_{k-i+2},r_2),$

$$\ldots, \sum_{r_j=r_{j0}}^{r_{j1}} g(n_{k-i+j},r_j)\} \quad j = 1,2,\ldots,i;$$

where

$$g(n;\ r) = \binom{n}{r}p_0^r(1-p_0)^{n-r}$$

$$r_{j0} = <c_j-r_1-r_2-\ldots-r_{j-1}>$$

$$r_{j1} = n_{k-i+j}$$

and $c_1,c_2,\ldots,c_j$ are defined by (2.12) and (2.10).

Proof: Conditioning on $Z_1 = r_1$ in (2.13) and following analogous arguments as those in Lemma 2.1, we obtain the result.

Corollary 2.2. If for given P*, the constants $\underset{\sim}{d} = (d_1,d_2,\ldots,d_k)$ associated with $\delta_1(\underset{\sim}{d})$ are so chosen that $b_{i,i}(\underset{\sim}{c}) \geq P^*$ for all $i = 1,2,\ldots,k$, then $P_{\underset{\sim}{p}}(CS|\delta_1(\underset{\sim}{d}) \geq P^* \quad \forall \underset{\sim}{p} \in \Omega$, where $\underset{\sim}{c}$ and $b_{i,i}(\underset{\sim}{c})$ are defined by (2.12) and (2.13), and $d_{k-i+1} = \alpha_i$, $i = 1,2,\ldots,k$.

Proof: The result follows from (2.3), (2.11) and (2.13).

2.2. Selection Procedure δ_2 and Conditional Selection Procedure δ_3; Unknown p_0

For simplicity, we assume $m = 1$ and $n_0 = n_1 = \ldots = n_k = n$. When p_0 is unknown, we propose the following procedures.

$\delta_2(\underset{\sim}{x};\ \underset{\sim}{u}) = a_{\epsilon_1(\underset{\sim}{x};\ \underset{\sim}{u})}$ where

$$\epsilon_1(\underset{\sim}{x};\ \underset{\sim}{u}) = \min\{i|\hat{X}_{i,k} \geq \bar{X}_0-u_i\}$$

where

$\hat{X}_{i,k}$ is defined by (1.1) and (1.2).

We also propose a conditional procedure δ_3 as follows.

$$\delta_3(\underset{\sim}{x};\ \underset{\sim}{v}|\underset{\sim}{t}) = a_{\epsilon_2(\underset{\sim}{x};\ \underset{\sim}{v}|\underset{\sim}{t})} \quad \text{where}$$

$$\epsilon_2(\underset{\sim}{x};\ \underset{\sim}{v}|\underset{\sim}{t}) = \min\{i|\hat{X}_{i,k} \geq \bar{X}_0 - v_i \text{ given that } \sum_{j=0}^{i} X_j = t_i\}$$

where

$$\underset{\sim}{t} = (t_1, t_2, \ldots, t_k),\ t_j = \sum_{0}^{j} X_r,\ j = 1,2,\ldots,k.$$

We note that when $(\bar{X}_0, \bar{X}_1, \ldots, \bar{X}_k)$ is observed, $\underset{\sim}{t}$ is automatically known. However, in some situations, the experimenter may have values of $\hat{X}_{i,k} - \bar{X}_0$ without knowing $(\bar{X}_0, \bar{X}_1, \ldots, \bar{X}_k)$. For example, the given data might be $\bar{X}_1 - \bar{X}_0,\ \bar{X}_2 - \bar{X}_0, \ldots, \bar{X}_k - \bar{X}_0$.

For our convenience, we denote δ_2 and δ_3, respectively, by $\delta_2(\underset{\sim}{u})$ and $\delta_3(\underset{\sim}{v}|\underset{\sim}{t})$ when there is no confusion.

For fixed i, and any $\underset{\sim}{p} \in \Omega_i$

$$P_{\underset{\sim}{p}}(CS|\delta_2(\underset{\sim}{u})) = P_{\underset{\sim}{p}}\Big(\bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^{j} (\hat{\hat{X}}_r \geq \bar{X}_0 - u_j)\Big)$$

$$\geq P_{\underset{\sim}{p}}\{\hat{\hat{X}}_{k-i+1} \geq \bar{X}_0 - u_{k-i+1}\}.$$

For fixed p_0, the right hand side is an increasing function of p_{k-i+1} keeping all p_{k-i+j} $(2 \leq j \leq i)$ fixed. By the same arguments as in the last section, we see that the right hand side attains its minimum at $p_{k-i+1} = p_{k-i+2} = \cdots = p_k = p_0$. By choosing $\underset{\sim}{p}_0$ defined by (2.2), and applying the analogous arguments, we conclude that

(2.14) $$\inf_{\Omega_i} P_{\underset{\sim}{p}}(CS|\delta_2(\underset{\sim}{u})) = \inf_{0\leq p_0\leq 1} P\{Y_1 \geq Y_0-w_1,\ Y_1+Y_2 \geq 2(Y_0-w_2),$$

$$\ldots, Y_1+Y_2+\ldots+Y_i \geq i(Y_0-w_i)\}.$$

where $Y_0, Y_1, Y_2, \ldots, Y_i$ are iid such that Y_0 is $b(n; p_0)$ and

(2.15) $$w_j = jnu_i.$$

Now define

(2.16) $$A(i; t_i, u_i) = \{Y_1 \geq Y_0-w_1,\ Y_1+Y_3 \geq 2(Y_0-w_2), \ldots, Y_1+Y_2+\ldots+Y_i \geq i(Y_0-w_i),\ \sum_{j=0}^{i} Y_j = t_i\}$$

(2.17) $$B(i; u_i) = \{Y_1 \geq Y_0-w_1,\ Y_1+Y_2 \geq 2(Y_0-w_2), \ldots, Y_1+Y_2+\ldots+Y_i \geq i(Y_0-w_i)\}$$

where w_i is defined by (2.15). Then, we have

(2.18) $$P(B(i; u_i)) = \sum_{t_i=0}^{n(i+1)} P(B(i; u) | \sum_{j=0}^{i} Y_j = t_i) P(\sum_{j=0}^{i} Y_j = t_i)$$

$$= \sum_{t_i=0}^{n(i+1)} \frac{P(A(i; t_i, \underset{\sim}{u}))}{\binom{n(i+1)}{t_i} p_0^{t_i}(1-p_0)^{ni+n-t_i}} P(\sum_{j=0}^{i} Y_j = t_i).$$

If for any $0 \leq p_0 \leq 1$ and any t_i $(0 \leq t_i \leq (i+1)n)$

(2.19) $$P(A(i; t_i, u_i)) \geq P^*\binom{n(i+1)}{t_i} p_0^{t_i}(1-p_0)^{ni+n-t_i}$$

holds, then it follows from (2.14), (2.18) and (2.19) that

(2.20) $$\inf_{\Omega_i} P_{\underset{\sim}{p}}(CS|\delta_2(\underset{\sim}{u})) \geq P^*.$$

Define

(2.21) $$\psi_i(t,s,u) = \{(x_1, x_2, \ldots, x_i) | x_1 \geq s-nu,\ x_1+x_2 \geq 2s-2nu,$$

$$\ldots, \sum_{1}^{i-1} x_j \geq (i-1)s-(i-1)nu, \sum_{1}^{i} x_j = t\} \subset I^i$$

(2.22) $$q_i(t,s,u) = \sum_{\psi_i(t,s,u)} \binom{n}{x_1}\binom{n}{x_2}\cdots\binom{n}{x_i}$$

(2.23) $$\phi_i(t,u) = \{(x_0,x_1,x_2,\ldots,x_i) \mid x_1 \geq x_0-nu,\ x_1+x_2 \geq 2x_0-2nu$$
$$\ldots \sum_{j=1}^{i} x_j \geq ix_0-inu,\ \sum_{j=0}^{i} x_j=t\}$$

(2.24) $$g_i(t,u) = \sum_{\phi_i(t,u)} \binom{n}{x_0}\binom{n}{x_1}\cdots\binom{n}{x_i}.$$

Then, we have

(2.25) $$P(A(i;\ t_i,u))/\binom{ni+n}{t_i}p_0^{t_i}(1-p_0)^{ni+n-t_i} = g_i(t_i,u)/\binom{ni+n}{t_i}.$$

It follows from (2.19) and (2.25) that in order to find u_i so that $\delta_2(\underset{\sim}{u})$ satisfies the P*-condition, it suffices to find u_i such that for all t_i $(0 \leq t_i \leq n(i+1))$

(2.26) $$g_i(t_i,\ u_i) \geq P^*\binom{ni+n}{t_i}$$

holds.

To compute $g_i(t,u)$ for given t and u, we may apply the following theorem.

Define

(2.27) $$\xi_i(t,\alpha,\beta) = \{(x_1,x_2,\ldots,x_i) \mid x_1 \geq \alpha,\ x_1+x_2 \geq \alpha+\beta,\ldots, \sum_{1}^{i-1} x_j$$
$$\geq \alpha + (i-2)\beta,\ \sum_{1}^{i} x_j = t\} \subset I^i.$$

(2.28) $$\zeta_i(t,\gamma) = \{(x_0,x_1,\ldots,x_i) \mid x_1 \geq x_0-\gamma,\ x_1+x_2 \geq 2(x_0-\gamma),\ldots,$$
$$\sum_{1}^{i} x_j \geq i(x_0-\gamma),\ \sum_{0}^{i} x_j = t\}$$

(2.29) $$u_i(t,\alpha,\beta) = \sum_{\xi_i(t_i,\alpha,\beta)} \binom{n}{x_1}\binom{n}{x_2}\cdots\binom{n}{x_i}$$

(2.30) $$v_i(t,\gamma) = \sum_{\zeta_i(t,\gamma)} \binom{n}{x_0}\binom{n}{x_1}\cdots\binom{n}{x_i}.$$

Then, we have the following

Theorem 2.2: (i) $$u_i(t,\alpha,\beta) = \sum_{r_1=<\alpha>}^{n} \binom{n}{r_1}\Big\{ \sum_{r_2=<\alpha+\beta-r_1>} \binom{n}{r_2}\Big\{\ldots$$

$$\Big\{ \sum_{r_j=\lambda_j}^{n} \binom{n}{r_j}\ldots\Big\{ \sum_{r_{i-1}=\lambda_{i-1}}^{\lambda} \binom{n}{r_{i-1}}\binom{n}{t-r_1-r_2-\ldots-r_{i-1}}\Big\}\Big\}\Big\}\Big\},$$

where

$$\lambda_j = <\alpha + (j-1)\beta-r_1-r_2-\ldots-r_{j-1}> \quad j = 2,3,\ldots,i-1,$$

$$\lambda = \min\{n,t-r_1-r_2-\ldots-r_{i-2}\}$$

(ii) $$v_i(t,\gamma) = \sum_{s=s_1}^{s_2} \binom{n}{s}u_i(t-s,\alpha,s-\gamma) \quad \text{where}$$

$$s_1 = \max\{0,t-in\}$$

$$s_2 = \min\{n,t, [\tfrac{t+i\gamma}{i+1}]\}$$

and $u_i(t-s,\alpha,s-\gamma)$ is given by (i).

Proof: To show (i), we condition on $x_1 = r_1$ in the set $\xi_i(t,\alpha,\beta)$, and note that the lower bound or r_1 is at least α. Thus we obtain $u_i(t,\alpha,\beta) = \sum_{r_1=<\alpha>}^{n} \binom{n}{r_1}u_{i-1}(t-r_1, \alpha+\beta-r_1,\beta)$. Using mathematical induction and noting that $u_2(t,\alpha,\beta) = \sum_{r=<\alpha>}^{r_0} \binom{n}{r}\binom{n}{t-r}$ where r_0 should exceed neither n nor t-s, we thus obtain (i).

To show (ii), we condition on the value of x_0 by taking $x_0 = s$ in the set $\zeta_i(t,\gamma)$. Then, in the set $\zeta_i(t,\gamma)$, we have simultaneously the conditions $\sum_1^i x_j \geq i(s-\gamma)$ and $\sum_1^i x_j = t-s$. In order that $\zeta_i(t,\gamma)$ is non-empty, we should have $t-s \geq i(s-\gamma)$ and this determines the upper bound of s. On the other hand, we also note that s should exceed neither n nor t and this concludes the value s_2 in (ii). For the lower bound, we note that s should not be less than t-in if this is positive. Finally, we note that the two conditions $\Sigma x_j \geq i(s-\gamma)$ and $\sum_1^i x_j = t-s$ combine into $\sum_1^i x_j = t-s$ and it concludes (ii).

This completes the proof of the theorem.

Now, by taking $\alpha = s-nu$ and $\beta = s-nu$, $\xi_i(t,s-nu,s-nu) = \psi_i(t,s,u)$; also, taking $\gamma = nu$ we have $\zeta_i(t,nu) = \phi_i(t,u)$. This concludes the following

<u>Corollary 2.3</u>: (i) $$q_i(t,s,u) = \sum_{r_1=<\alpha>}^{n} \binom{n}{r_1}\{ \sum_{r_2=<2\alpha-r_1>}^{n} \binom{n}{r_2}\{ \ldots$$

$$\{ \sum_{r_j=\lambda_j}^{n} \binom{n}{r_j} \ldots \{ \sum_{r_{i-1}=\lambda_{i-1}}^{\lambda} \binom{n}{r_{i-1}} \binom{n}{t-r_1-r_2-\ldots-r_{i-1}} \}\}\}\},$$

where

$$\alpha = s-nu$$

$$\lambda_j = <j\alpha-r_1-r_2-\ldots-r_{j-1}>, \qquad j = 1,2,\ldots,i-1.$$

$$\lambda = \min\{n,\ t-r_1-r_2-\ldots-r_{i-2}\}.$$

(ii) $$g_i(t,u) = \sum_{s=s_1}^{s_2} \binom{n}{s} q_i(t-s,s,u)$$

where s_1 and s_2 are defined in (ii) of Theorem 2.2 by taking $\gamma = nu$.

<u>Theorem 2.3</u>. (i) If, for given $P^*(\frac{1}{k+1} < P^* < 1)$,

$g_i(t_i,u_i) \geq P^*\binom{ni+n}{t_i}$ observing that $\underset{\sim}{t} = (t_1,t_2,\ldots,t_k)$, then, $\delta_3(\underset{\sim}{v}|\underset{\sim}{t})$ satisfies the P*-condition, where $v_{k-i+1} = u_i$, $i = 1,2,\ldots,k$.

(ii) If, for some given $P^*(\frac{1}{k+1} < P^* < 1)$, $g_i(t_i,u_i) \geq P^*\binom{ni+n}{t_i}$ for all $t_i = 0,1,2,\ldots,(i+1)n$ and $i = 1,2,\ldots,k$, then $\delta_2(\underset{\sim}{u}')$ satisfies the P*-condition where $u'_{k-i+1} = \max_{0\leq t\leq(i+1)n} u_i(t)$.

Proof: It follows from (2.14), (2.16), (2.25) and the definition of $\delta_3(\underset{\sim}{v}|\underset{\sim}{t})$ that (i) holds. For (ii), we note that $P(A(i;\, t_i,u_i))$ (defined by (2.16)) is increasing in u_i and (2.26) holds for all $t_i = 0,1,\ldots,(i+1)n$. Finally, it follows from (2.14), (2.18) and (2.25). Finally, we define

$$\xi'_i(t;\, \alpha_1,\alpha_2;\, \beta,\gamma) = \{(x_1,x_2,\ldots,x_i)|\alpha_1 \leq x_1 \leq \alpha_2,\ \alpha_1+\beta \leq x_1+x_2 \leq \alpha_2+\gamma,$$
$$\ldots,\alpha_1+(i-2)\beta \leq \sum_1^{i-1} x_j \leq \alpha_2+(i-2)\gamma,\ \sum_1^{i} x_j = t\},$$

$$\zeta'_i(t;\, \gamma,\delta) = \{(x_0,x_1,\ldots,x_i)|x_0-\gamma \leq x_1 \leq x_0+\delta, 2(x_0-\gamma) \leq x_1+x_2 \leq$$
$$2(x_0+\delta),\ldots,i(x_0-\gamma) \leq \sum_1^{i} x_j \leq i(x_0+\delta),\ \sum_0^{i} x_r = t\},$$

$$u'_i(t;\, \alpha_1,\alpha_2;\, \beta,\gamma) = \sum_{\xi'_i(t;\, \alpha_1,\alpha_2;\beta,\gamma)} \binom{n}{x_1}\binom{n}{x_2}\cdots\binom{n}{x_i},$$

$$v'_i(t;\, \gamma,\delta) = \sum_{\zeta'_i(t;\, \gamma,\delta)} \binom{n}{x_0}\binom{n}{x_1}\cdots\binom{n}{x_i}.$$

Using analogous arguments, we obtain the following

<u>Corollary 2.4</u>: (i) $u'_i(t;\, \alpha_1,\alpha_2;\, \beta,\gamma) = \sum_{r_1=\langle\alpha_1\rangle}^{[\alpha_2]} \binom{n}{r_1}\{\sum_{r_2=s_2}^{s'_2} \binom{n}{r_2}\{\ldots$

$$\{\sum_{r_j=s_j}^{s'_j} \binom{n}{r_j}\ldots\{\sum_{r_{i-i}=s_{i-1}}^{\lambda} \binom{n}{r_{i-1}}\binom{n}{t-r_1-r_2-\ldots-r_{i-1}}\}\}\}\}$$

where

$$s_j = \langle\alpha_1+(j-1)\beta-r_1-r_2-\cdots-r_{j-1}\rangle, \qquad j = 2,3\ldots,i-1,$$

$$s_j' = [\alpha_2+(j-1)\gamma-r_1-r_2-\cdots-r_{j-1}], \qquad j = 2,3\ldots,i-1,$$

$$\gamma = \min\{n,\ t-r_1-r_2-\cdots-r_{i-2}\}.$$

(ii) $$v_i'(t;\gamma,\delta) = \sum_{s=s_0}^{s_1} \binom{n}{s} u_i'(t-s;s-\gamma,s+\delta;s-\gamma,s+\delta)$$

where

$$s_0 = \max\{[\tfrac{t+i\gamma}{i+1}],\ t-in,0\}$$

$$s_1 = \min\{\langle\tfrac{t-i\delta}{i+1}\rangle,\ n,t\}.$$

Computations of $v_i(t_i)$ and u_i'. For given n, i, P* and t_i $(0 \le t_i \le n(i+1))$, we start with u = 0, and apply Corollary 2.3 to compute $g_i(t_i,0)$. If $g_i(t_i,0) \ge P^*\binom{ni+1}{t_i}$, then stop and take $u_i(t_i) = 0$; otherwise, increase u by an amount of $\frac{1}{in}$. Again, using recurrence relation, compute $g_i(t_i,u)$. If $g_i(t_i,u) \ge P^*\binom{in+n}{t_i}$, then we take $u_i(t_i) = u$; otherwise, we continue this process until for the first time $g_i(t_i,u) \ge P^*\binom{in+n}{t_i}$. Then, we take $v_{k-i+1}(t_{k-i+1}) = u_i(t_{k-i+1})$. For the procedure δ_1, we take $u_i' = \max_{0\le t_i\le(i+1)n} u_i(t_i)$. Values of $u_i(t_i)$ for n = t(1)10, P* = 0.90,0.95 and i = 1(1)5 are given in Table II.

SOME COMMENTS ON THE CONDITIONAL ISOTONIC RULE δ_2

As it can be seen, the unconditional selection procedure δ_2 defined in Section 2.2 always satisfied the P*-condition. As a matter of fact, it follows from (2.14) that the infimum of the probability of a correct selection attains 1 if $u_i = 1$. On the other hand, when the condition of total sum of observations is imposed on the event $B(i;\ u_i)$ defined by (2.17), it becomes the event $A(i;\ t_i,u_i)$ defined by (2.16), and the probability of $A(i;\ t_i,u_i)$ decreases. It follows from (2.18) and (2.25) that

$$(3.1)\qquad P(B(i;u)) = \sum_{t_i=0}^{n(i+1)} \frac{g_i(t_i,u)}{\binom{ni+n}{j}} P\left(\sum_{j=0}^{i} Y_j = t_i\right)$$

where $Y_0,Y_1,\ldots,Y_i$ are iid such that Y_0 is $b(n;p)$ for some unknown $0 < p < 1$. Let

$$(3.2)\qquad h_i(j;\ u) = g_i(j,u)/\binom{ni+n}{j}, \qquad j = 0,1,2,\ldots,(i+1)n$$

where

$$0 < u < 1.$$

Then, (3.1) becomes

$$(3.3)\qquad P(B(i;\ u)) = \sum_{j=0}^{(i+1)n} h_i(j;\ y)\cdot b(j;\ (i+1)n,p)$$

where $b(j,\ n;\ p)$ is the probability of the event $\sum_{0}^{i} Y_r = j$. If $h_i(j;\ y) \geq P^*$ for all $j = 0,1,2,\ldots,(i+1)n$ and for some $0 < u < 1$, then $P(B(i;\ u)) \geq P^*$. However, it is not true that $h_i(j;\ u) \geq P^*$ for all j, when the right hand side of (3.3) is not less than P^*. As some computations show, for some j (when j is large), $h_i(j;\ u)$ never reaches P^* (e.g. $P^* = 0.95$), no matter how large u is. This undesirable situation fortunately, never occurs in the conditional selection procedure proposed in [3].

Table I. Values of d_1, d_2, d_3, d_4 for $P^* = 0.90$ (upper entry) and $P^* = 0.95$ (lower entry) for the procedure δ_1.*

	d_1-values				d_2-values			
n \ p_0	0.2	0.3	0.4	0.5	0.2	0.3	0.4	0.5
5				0.300 -			0.350 0.400	0.350 -
6				0.333 -			0.233 0.400	0.333 -
7				0.214 0.357			0.293 -	0.250 0.357
8			0.275 -	0.250 -		0.175 0.300	0.275 -	0.250 -
9			0.178 0.289	0.167 0.278		0.217 0.272	0.206 0.317	0.250 0.306
10			0.200 -	0.200 0.300		0.225 -	0.225 0.275	0.225 0.300
11		0.209 -	0.218 -	0.227 -		0.232 -	0.241 -	0.250 -
12		0.133 0.217	0.150 0.233	0.167 0.250	0.158 0.200	0.175 0.238	0.213 0.254	0.188 0.271
13		0.146 0.223	0.169 0.246	0.192 -	0.123 0.200	0.165 0.242	0.188 0.265	0.212 0.250
14		0.157 -	0.186 -	0.143 0.214	0.146 0.200	0.157 0.193	0.186 -	0.179 0.214
15		0.167 -	0.133 0.200	0.167 0.233	0.133 -	0.183 -	0.183 0.217	0.183 0.250
16		0.113 0.175	0.150 0.213	0.186 -	0.138 -	0.175 -	0.150 0.213	0.188 -
17	0.141 -	0.124 0.182	0.165 -	0.147 0.206	0.141 -	0.138 0.197	0.179 0.209	0.162 0.221
18	0.089 0.144	0.133 0.189	0.122 0.178	0.167 -	0.158 -	0.147 0.203	0.164 0.192	0.181 0.208
19	0.095 0.147	0.142 -	0.137 0.189	0.132 0.184	0.134 0.161	0.155 0.182	0.150 0.203	0.145 0.197
20	0.100 0.150	0.150 -	0.150 0.200	0.150 0.200	0.113 0.163	0.150 -	0.150 0.200	0.150 0.200

*The entries in this table are the (possibly) the smallest d_i-values satisfying $a_i(d_i) \geq P^*$. The missing entries in the above table are equal to the corresponding p_0 values. For $p_0 = 0.1$, $d_1 = d_2 = 0.1$ for all n. A "-" means the same value as the preceding value in the same column n.

Table I (continued)

p_0 / n	d_3-values				d_4-values			
	0.2	0.3	0.4	0.5	0.2	0.3	0.4	0.5
5			0.300 0.400	0.300 0.367			0.350 0.400	0.350 0.400
6			0.261 0.400	0.361 -			0.254 0.400	0.333 -
7			0.257 -	0.214 0.357			0.257 -	0.250 0.357
8		0.175 0.300	0.296 -	0.271 -		0.175 0.300	0.275 -	0.250 -
9		0.189 0.263	0.196 0.307	0.241 0.296		0.217 0.272	0.192 0.303	0.250 0.306
10		0.217 -	0.217 0.267	0.217 0.317		0.225 -	0.200 0.250	0.225 0.300
11		0.224 -	0.233 -	0.242 -		0.232 -	0.230 -	0.250 -
12	0.172 0.200	0.189 0.231	0.206 0.247	0.181 0.264	0.158 0.200	0.175 0.238	0.192 0.233	0.188 0.271
13	0.123 0.200	0.146 0.223	0.169 0.246	0.192 0.244	0.123 0.200	0.165 0.242	0.179 0.256	0.212 0.250
14	0.129 0.200	0.169 0.217	0.198 -	0.190 0.226	0.146 0.200	0.157 0.211	0.186 -	0.179 0.214
15	0.136 0.156	0.167 -	0.167 0.200	0.167 0.233	0.133 0.167	0.167 -	0.167 0.217	0.183 0.250
16	0.148 -	0.175 -	0.150 0.213	0.188 -	0.138 -	0.175 -	0.150 0.213	0.188 -
17	0.141 -	0.124 0.182	0.165 0.194	0.147 0.206	0.141 -	0.138 0.190	0.172 0.209	0.163 0.221
18	0.154 -	0.143 0.198	0.187 -	0.176 0.213	0.158 -	0.147 0.196	0.185 0.192	0.181 0.208
19	0.130 0.156	0.151 0.177	0.146 0.198	0.158 0.193	0.134 0.161	0.155 0.175	0.143 0.203	0.171 0.197
20	0.108 0.158	0.158 -	0.150 0.200	0.150 0.200	0.123 0.163	0.150 -	0.150 0.200	0.150 0.200

The missing entries in the above table are equal to the corresponding p_0 values. For $p_0 = 0.1$, $d_3 = d_4 = 0.1$ for all n. A "-" means the same value as the preceding value in the same column.

Table II. Values of $u_i(t_i)$, $i = 1,\ldots,5$ for $P^* = 0.90$ (upper entry) and $P^* = 0.95$ (lower entry).

N = 5

i/t	1	2	3	4	5	6	7	8	9	10
1	0.400 -	0.600 -	0.400 -	0.600 -	0.800 -					
2	0.300 -	* 0.500	0.500 *	* 0.600	* 0.500	* *	* 0.700			
3	0.267 -	* 0.467	* -	* -	* -	* -	0.400 -	0.467 -	0.667 -	
4	0.250 -	* -	0.150 -	* -	* -	0.250 -	* -	* -	0.450 -	0.850 -
5	0.240 -	* -	* -	* -	* -	* -	* -	* -	* -	* -

N = 6

i/t	1	2	3	4	5	6	7	8	9	10
1	0.333 -	0.500 -	0.333 0.667	0.500 -	0.667 -	0.500 -				
2	0.250 -	* 0.417	0.417 -	* 0.500	* 0.583	* *	0.500 0.583	0.583 *	* 0.833	
3	0.222 -	* 0.389	0.333 *	0.278 -	0.222 -	* -	0.333 -	0.278 0.389	0.389 -	* -
4	0.208 -	* -	0.125 *	* -	* -	* -	0.208 -	* -	* -	* 0.375
5	0.200 -	* -	* 0.367	* -	* -	* -	* -	* -	* -	* -

N = 7

i/t	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.286 -	0.429 -	0.286 0.571	0.429 -	0.571 -	0.429 -	0.571 -							
2	0.214 -	* 0.357	0.357 -	* 0.429	* 0.500	* *	0.429 0.500	0.357 -	0.357 0.500	0.500 -				
3	0.190 -	* 0.333	0.286 *	0.238 *	0.190 -	* -	* 0.286	0.238 -	0.190 0.333	0.333 -	* -	* 0.429	0.476 0.619	
4	0.179 -	* -	0.107 -	* -	0.071 *	0.036 0.071	* 0.107	0.107 -	* 0.179	0.179 -	* -	0.321 -	0.464 -	0.750 -

The entries of this table are the $u_i(t_i)$-values satisfying $g_i(t_i,u_i) \geq P^*(\frac{(i+1)n}{t_i})$ with $t_i = \sum_{j=0}^{i} x_j$. A "*" means the same as the preceding value in the same row and a "-" means the same value as the preceding value in the same column. The constant $v_i \equiv v_i(t_i)$ needed for the rule δ_3 is given by $v_i(t_i) = u_i(t_i)$.

Table II (continued)

N = 8

i/t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0.125 -	0.250 -	0.125 0.375	0.250 -	0.375 -	0.250 0.500	0.375 -	0.250 0.500										
2	0.125 -	0.250 -	* -	* 0.313	* 0.375	* *	0.313 *	0.375 0.433	* 0.375	* -	* -	0.500 0.625						
3	0.167 -	* 0.292	0.250 *	0.292 -	* -	0.167 *	0.250 -	0.208 0.292	0.167 *	0.292 -	* -	* -	* 0.333	0.417 -	0.542 -			
4	0.125 -	0.094 *	0.063 0.094	0.063 -	0.031 *	0.031 -	* -	0.063 -	* -	* -	0.125 -	* -	* -	0.250 -	0.375 -	0.500 0.625		
5	0.150 -	* -	* 0.275	0.225 *	0.200 *	0.175 0.200	* *	0.150 *	* 0.175	* -	0.150 -	* 0.175	* -	* -	* -	* 0.200	0.200 0.275	0.275 -

N = 9

i/t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0.222 -	0.333 -	0.444 -	0.333 -	0.444 -	0.333 0.555	0.444 -	0.333 -	0.444 -								
2	0.167 -	0.278 -	* -	* 0.389	* *	* *	0.333 *	0.389 0.444	* -	0.333 *	0.278 *	0.389 -	* 0.500				
3	0.148 -	* 0.259	0.222 *	* -	* 0.296	* -	0.222 *	0.185 *	0.259 -	* -	0.222 0.259	* -	* -	* -	* -	0.370 -	0.481 0.592

N = 10

i/t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0.200 -	0.300 -	0.400 -	0.300 -	0.400 -	0.300 -	0.400 -	0.300 0.500	0.400 -	0.300 0.500										
2	0.150 -	0.250 -	* -	* 0.350	* *	0.350 *	0.300 *	0.350 0.400	* 0.350	* 0.450	* 0.400	* 0.350	* *	* 0.450	0.450 0.650					
3	0.133 -	* 0.233	0.200 *	0.233 -	* 0.267	* -	* -	* -	* -	* -										
4	0.125 -	0.100 -	0.075 -	0.050 -	0.025 -	* -	* -	* -	* 0.050	* 0.025	0.233 -	* -	* -	* -	* -	* 0.300	0.267 0.333	0.333 -	0.433 0.533	
5	0.120 -	* -	* 0.220	0.180 0.220	0.220 -	* -	* -	* -	* -	* -	0.180 0.220	0.160 0.220	* *	0.140 *	0.120 0.160	* 0.180	* 0.160	0.140 -	0.120 *	* 0.160

The entries of this table are the $u_i(t_i)$-values satisfying $g_i(t_i,u_i) \geq P^*(\frac{(i+1)n}{t_i})$ with $t_i = \sum_{j=0}^{i} x_j$. A "*" means the same as the preceding value in the same row and a "-" means the same value as the preceding value in the same column. The constant $v_i \equiv v_i(t_i)$ needed for the rule δ_3 is given by $v_i(t_i) = u_i(t_i)$.

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