ON THE DISTRIBUTION OF THE STUDENTIZED MAXIMUM OF EQUALLY CORRELATED NORMAL RANDOM VARIABLES*

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ABSTRACT

Let $X_1,\ldots,X_k$ have a joint k-variate normal distribution with zero means, common unknown variance σ^2 and known correlation matrix (ρ_{ij}), where $\rho_{ij} = \rho$ for all $i \neq j$. Let s^2 be distributed independently of the X_i such that $\nu s^2/\sigma^2$ has a chi-squared distribution with ν degrees of freedom. New tables with wider coverage and more accuracy that the previously published ones are given for the percentage points of $Y = \max_{1\leq i\leq k} \frac{X_i}{s}$. Some basic theoretical results are given in Section 2. The next section describes Hartley's results for approximating the distribution function of Y. Besides a brief review of existing tables (Section 4), the paper discusses the construction of new tables based on Hartley's results (Section 5) and some specific applications (Section 6).

Key words and phrases: Normal variables; Equicorrelated; Studentized; Maximum; Multivariate t; Percentage points; Tables; Subset selection procedures; Multiple comparisons; Prediction intervals; Tests of hypotheses.

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1. INTRODUCTION

Let $X_1,\ldots,X_k$ have a joint k-variate normal distribution with zero means, a common unknown variance σ^2 and a known correlation matrix $\Omega = (\rho_{ij})$. Also, let s^2 be a random variable independent of the X_i's such that $\nu s^2/\sigma^2$ has a chi-square distribution with ν degrees of freedom. Then the joint distribution of $t_1,t_2,\ldots,t_k$, where $t_i = X_i/s$, is a (central) multivariate t distribution which has the density (see Cornish [2] or Dunnett and Sobel [4])

$$(1.1)\qquad g(t_1,\ldots,t_k) = \frac{|A|^{\frac{1}{2}}\Gamma[(k+\nu)/2]}{(\nu\pi)^{k/2}\Gamma(\nu/2)}\left[1+\frac{1}{\nu}\sum_i\sum_j a_{ij}t_it_j\right]^{-\left(\frac{\nu+k}{2}\right)}$$

where $A = (a_{ij}) = \Omega^{-1}$.

We are interested in the distribution of

$$(1.2)\qquad Y = \max t_i = \frac{\max X_i}{s}\,.$$

Of course, the percentage points of the distribution of Y are the same as the corresponding equi-percentage points of the k-variate t distribution in (1.1). These percentage points are needed in several problems of statistical inference such as tests for the multiple comparisons of means (or mean vectors) against one-sided alternatives under a general analysis

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of variance (or multivariate analysis of variance) model, simultaneous confidence intervals for multiple comparisons between several treatment means and a control mean, and selection and ranking procedures for selecting the best population and for selecting populations that are better than a control. So it is not surprising that several authors have provided tables of the percentage points of the distribution of Y. These tables differ with respect to the ranges of the various parameters, the accuracy of the values, the probability levels chosen, and the numerical methods employed in constructing these tables.

Besides providing new tables of these percentage points with wider coverage and more accuracy, our emphasis in this paper is on Hartley's method of approximating $\mathrm{pr}\{Y \leq y\}$ by a series in which the leading term corresponds to the case of $\nu = \infty$ (σ^2 known) and the successive terms are in powers of ν^{-1}. Hartley [11] obtained this as the solution of a certain differential-difference equation. His method is found very useful; however, no published tables of the percentage points of Y until now were based on this method.

In this paper, we discuss some basic theoretical results (Section 2) and Hartley's results with an additional correction term (Section 3). Besides reviewing briefly the existing tables (Section 4), we discuss the construction of the new tables (Section 5), and some specific applications (Section 6).

2. BASIC THEORETICAL RESULTS

In considering the distribution of Y we assume without loss of generality that $\sigma = 1$. Further, we only consider the case where $\rho_{ij} = \rho > 0$ for all $i \neq j$. In this case, it is well-known (Dunnett and Sobel [4]) that the random variables $X_1, X_2, \ldots, X_k$ can be generated from k+1 independent standard normal variables $Z_0, Z_1, \ldots, Z_k$ by setting

$$(2.1) \qquad X_i = (1-\rho)^{\frac{1}{2}} Z_i - \rho^{\frac{1}{2}} Z_0, \quad i = 1, \ldots, k.$$

Using this representation, we can write the distribution function $F_k(w;\, \rho)$ of $W = \max X_i$ in the form

$$(2.2) \qquad F_k(w;\, \rho) = \int_{-\infty}^{\infty} \Phi^k\left[\frac{w+\rho^{\frac{1}{2}}t}{(1-\rho)^{\frac{1}{2}}}\right] \varphi(t)dt$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ denote the density and the cdf of a standard normal variate. By a similar argument, the distribution function $G_{k,\nu}(y;\, \rho)$ of Y is given by

$$(2.3) \qquad G_{k,\nu}(y;\, \rho) = \int_0^{\infty} \int_{-\infty}^{\infty} \Phi^k\left[\frac{yu+\rho^{\frac{1}{2}}t}{(1-\rho)^{\frac{1}{2}}}\right]\varphi(t)h_\nu(u)dt\, du$$

where $h_\nu(\cdot)$ is the density function of s. In the sequel, unless the context demands us to be more specific, we will simply write $G_\nu(y)$ and $F(y)$ instead of $G_{k,\nu}(y;\, \rho)$ and $F_k(y;\, \rho)$, respectively. It is a well-known fact that $\lim_{\nu\to\infty} G_\nu(y) = F(y)$.

The moments of Y can be written as a product of two expectations. Let $Z_{[k]} = \max(Z_1, \ldots, Z_k)$. For $r < \nu$,

$$(2.4) \qquad E[Y^r] = E[s^{-r}]E[(\sqrt{1-\rho}\, Z_{[k]} - \sqrt{\rho}\, Z_0)^r]$$

$$= A_{-r} \sum_{j=0}^{[r/2]} \binom{r}{2j} \frac{(2j)!}{2^j j!} \rho^j (1-\rho)^{\frac{r-2j}{2}} a_{k,r-2j},$$

where $[r/2]$ is the largest integer less than or equal to $r/2$, $a_{k,i}$ is the ith moment of the largest of k independent standard normal variables, and

$$(2.5) \qquad A_\beta = E[s^\beta] = \frac{\Gamma(\frac{\nu+\beta}{2})}{(\frac{\nu}{2})^{\beta/2}\Gamma(\frac{\nu}{2})}, \qquad \beta > -\nu.$$

3. HARTLEY'S METHOD

Hartley [11] considers the statistic $U = W/S$, where W has a distribution involving a scale parameter σ, S is independent of W, and S^2/σ^2 has a chi-square distribution with ν degrees of freedom. In our problem, $W = \max X_i$ and $S = \sqrt{\nu}\, s$. We have taken $\sigma = 1$ without loss of generality. Thus $Y = \sqrt{\nu}\, U$. Let H_ν denote the distribution function of U. It has been shown by Hartley that

$$(3.1) \qquad u \frac{d}{du} H_\nu(u) = \nu[H_{\nu+2}(u) - H_\nu(u)].$$

Hartley's method of solving the differential-difference equation (3.1) can be described briefly as follows.

Regarding the degrees of freedom ν as a continuous second variable taking on positive values, we seek a function $H(\nu,u)$ of two variables for which

$$(3.2) \qquad u \frac{\partial H(\nu,u)}{\partial u} - \nu[H(\nu+2,u) - H(\nu,u)] = 0$$

and

$$(3.3) \qquad \lim_{\nu\to\infty} H(\nu,u) = F(u).$$

Equation (3.2) is now converted into a partial differential equation by expanding the finite difference in a Taylor series. Taking the expansion with two more additional terms than Hartley did, we get

$$(3.4) \quad u \frac{\partial H}{\partial u} - \nu[2 \frac{\partial H}{\partial \nu} + 2 \frac{\partial^2 H}{\partial \nu^2} + \frac{4}{3} \frac{\partial^3 H}{\partial \nu^3} + \frac{2}{3} \frac{\partial^4 H}{\partial \nu^4} + \frac{4}{15} \frac{\partial^5 H}{\partial \nu^5} + \frac{4}{45} \frac{\partial^6 \bar{H}}{\partial \nu^6}] = 0,$$

where the arguments of H are u and ν, and those of $\bar{H}$ are u and some mean value between ν and $\nu+2$. By introducing as new variables $y = \log u$ and $x = \log \nu$, the partial differential equation (3.4) is transformed into

$$(3.5) \quad \frac{\partial H}{\partial y} - 2\frac{\partial H}{\partial x} = 2e^{-x}\left(\frac{\partial^2 H}{\partial x^2} - \frac{\partial H}{\partial x}\right) + \frac{4}{3}e^{-2x}\left(\frac{\partial^3 H}{\partial x^3} - 3\frac{\partial^2 H}{\partial x^2} + 2\frac{\partial H}{\partial x}\right) +$$

$$\frac{2}{3}e^{-3x}\left(\frac{\partial^4 H}{\partial x^4} - 6\frac{\partial^3 H}{\partial x^3} + 11\frac{\partial^2 H}{\partial x^2} - 6\frac{\partial H}{\partial x}\right) +$$

$$\frac{4}{15}e^{-4x}\left(\frac{\partial^5 H}{\partial x^5} - 10\frac{\partial^4 H}{\partial x^4} + 35\frac{\partial^3 H}{\partial x^3} - 50\frac{\partial^2 H}{\partial x^2} + 24\frac{\partial H}{\partial x}\right) +$$

$$\frac{4}{45}e^{-5x}\left(\frac{\partial^6 \bar{H}}{\partial x^6} - 15\frac{\partial^5 \bar{H}}{\partial x^5} + 85\frac{\partial^4 \bar{H}}{\partial x^5} - 225\frac{\partial^3 \bar{H}}{\partial x^3} +\right.$$

$$\left.274\frac{\partial^2 \bar{H}}{\partial x^2} - 120\frac{\partial \bar{H}}{\partial x}\right),$$

where the arguments of H are y and x, and those of $\bar{H}$ are y and some mean value between x and $\log(e^x + 2)$.

Hartley obtains an approximate solution to (3.5) for large and moderate values of x with the help of an iterative process. When $x \to \infty$, the equation (3.5) tends to the limiting form $\frac{\partial H}{\partial y} - 2\frac{\partial H}{\partial x} = 0$ to which Hartley obtains the solution $H_0(x,y) = F(e^{y+\frac{1}{2}x})$. This provides the first approximation. Expressed in terms of u and ν, H_0 depends on $\sqrt{\nu}\, u$. This means that, for large ν, $G_\nu(u) = H_\nu(\frac{u}{\sqrt{\nu}})$ is equal to $F(u)$ to a first approximation (which is independent of ν). Successive closer approximations are obtained by Hartley iteratively. His method of solving the non-homogeneous partial differential equation $\frac{\partial H}{\partial y} - 2\frac{\partial H}{\partial x} = \psi(y,x)$ uses the theory of characteristics. For details of the steps from this point on, we refer to Hartley's paper. We will just give here the results of Hartley for the three correction terms involving ν^{-1}, ν^{-2}, and ν^{-3}, and add a fourth term involving ν^{-4} which is derived by carrying out one more iteration. The details of derivation are not given here.

Let

(3.6) $$G_{\nu,m}(y) = F(y) + \sum_{i=1}^{m} L_i(y)$$

where L_i is the ith correction term. Also, let $F^{(j)}(y)$ denote the jth order derivative of F(y) w.r.t. y. Then the first four correction terms can be written as

(3.7)
$$L_1(y) = \frac{1}{\nu}[\alpha^{(2)} - \alpha^{(1)}],$$
$$L_2(y) = \frac{1}{6\nu^2}[3\alpha^{(4)} - 10\alpha^{(3)} + 9\alpha^{(2)} - 2\alpha^{(1)}],$$
$$L_3(y) = \frac{1}{6\nu^3}[\alpha^{(6)} - 7\alpha^{(5)} + 17\alpha^{(4)} - 17\alpha^{(3)} + 6\alpha^{(2)}],$$
$$L_4(y) = \frac{1}{360\nu^4}[15\alpha^{(8)} - 180\alpha^{(7)} + 830\alpha^{(6)} - 1848\alpha^{(5)} + 2015\alpha^{(4)} - 900\alpha^{(3)} + 20\alpha^{(2)} + 48\alpha^{(1)}],$$

where

$$\alpha^{(j)} = \frac{1}{2^j}\phi^{(j)}(y), \quad j = 1,2,\ldots,8,$$

(3.8)
$$\phi^{(1)}(y) = y\,F^{(1)}(y),$$
$$\phi^{(2)}(y) = y^2F^{(2)}(y) + y\,F^{(1)}(y),$$
$$\phi^{(3)}(y) = y^3F^{(3)}(y) + 3y^2F^{(2)}(y) + y\,F^{(1)}(y),$$
$$\phi^{(4)}(y) = y^4F^{(4)}(y) + 6y^3F^{(3)}(y) + 7y^2F^{(2)}(y) + y\,F^{(1)}(y),$$
$$\phi^{(5)}(y) = y^5F^{(5)}(y) + 10y^4F^{(4)}(y) + 25y^3F^{(3)}(y) + 15y^2F^{(2)}(y) + y\,F^{(1)}(y),$$
$$\phi^{(6)}(y) = y^6F^{(6)}(y) + 15y^5F^{(5)}(y) + 65y^4F^{(4)}(y) + 90y^3F^{(3)}(y) + 31y^2F^{(2)}(y) + y\,F^{(1)}(y),$$
$$\phi^{(7)}(y) = y^7F^{(7)}(y) + 21y^6F^{(6)}(y) + 140y^5F^{(5)}(y) + 350y^4F^{(4)}(y) + 301y^3F^{(3)}(y) + 63y^2F^{(2)}(y) + y\,F^{(1)}(y),$$

$$\begin{aligned}\phi^{(8)}(y) &= y^8F^{(8)}(y) + 28y^7F^{(7)}(y) + 266y^6F^{(6)}(y) + \\ &\quad 1050y^5F^{(5)}(y) + 1701y^4F^{(4)}(y) + \\ &\quad 966y^3F^{(3)}(y) + 127y^2F^{(2)}(y) + y\,F^{(1)}(y).\end{aligned} \tag{3.8 cont.}$$

The correction term $L_3(y)$ given without the details of derivation by Hartley in terms of the $\phi^{(j)}$ is found to be in error; stated in terms of the $\alpha^{(j)}$, his coefficient of $\alpha^{(3)}$ is -22/3 instead of -17.

Now, it is easy to see that

$$F^{(j)}(y) = \frac{1}{\rho^{j/2}}\int_{-\infty}^{\infty}\Phi^k\left[\frac{\sqrt{\rho}t+y}{\sqrt{1-\rho}}\right]H_j(t)\varphi(t)dt, \tag{3.9}$$

where $H_j(t)$ is the Hermite polynomial in t of degree j. Thus the evaluation of $G_{\nu,m}(y)$ in (3.6) involves the evaluation of the integral in (3.9) for $j = 0,1,\ldots,8$.

Before we discuss the details of our numerical evaluation of the percentage points of the distribution of Y by using the approximation $G_{\nu,m}(y)$ to its distribution function $G_\nu(y)$, we will give a brief account of the existing tables.

4. EXISTING TABLES: BRIEF ACCOUNT

As we noted earlier, several authors have discussed the distribution of Y and have provided relevant tables. We give a brief description of these tables.

Tables in the case of $\nu = \infty$ (i.e. the percentiles of W) have been given by Bechhofer [1], Gupta [6], Gupta, Nagel and Panchapakesan [8], and Milton [14]. Bechhofer's table gives several percentage points (ranging from 5% to 99.95%) to four decimal places for $k = 1(1)9$ and $\rho = 0.5$. Gupta [6] has tabulated the 75, 90, 95, 97.5, and 99 percentage points to three decimal places for $\rho = 0.5$ and $k = 1(1)50$. Gupta [6] has also given the values of $F_k(w;\ \rho)$ to five decimal places for $k = 1(1)12$, $w = -3.5(0.1)3.5$ and seventeen selected values of ρ ranging from 0.1 to 0.9. The values of $F_k(w;\ \rho)$ have also been provided by Milton [14] to eight decimal places for $\rho = 0.5$, $k = 2(1)9(5)24$, and $w = 0.00(0.05)5.15$. Finally, the table of Gupta, Nagel and Panchapakesan [8] gives the 75, 90, 95, 97.5 and 99 percentage points for $k = 1(1)10(2)50$ and the seventeen values of ρ chosen by Gupta [6].

Percentage points of Y have been earlier tabulated by several authors. Pillai and Ramachandran [15] have tabulated the 95 percent points to two decimal places for $\rho = 0$, $k = 1(1)8$, and $\nu = 3(1)10,12,14,15,16,18,20,24,30,40,60,120$ and ∞.

Dunnett and Sobel [4] have considered only the bivariate t (k=2) and tabulated the percentage points (three decimal places) as well as the values of $G_{k,\nu}(y)$ (five decimal places) for $\rho = \pm\ 0.5$ and $\nu = 1(1)30(3)60(15)120,150,300,600,\infty$. Their table of the values of $G_{k,\nu}(y)$ covers $y = 0.00(0.25)2.50(0.50)10.00$ and the percentage levels chosen for the other table are 50, 75, 90, 95 and 99.

Dunnett [3] and Gupta and Sobel [9] have considered the case of $\rho = 0.5$ and $k \geq 2$. Dunnett's table gives the 95 and 99 percent points to two decimal places for $k = 1(1)9$ and $\nu = 5(5)25$ whereas in the Gupta-Sobel table (correct to two decimal places) $k = 2,5,10(1)16,18,20(5)40,50$; $\nu = 15(1)20,24,30,36,40,48,60,80,100,120,360,\infty$ and the percentage levels are 75, 90, 95, 97.5, 99, 99.75, 99.9, and 99.95. Dunnett's method is to numerically evaluate $G_{k,\nu}(y)$ for three successive values of y differing by 0.1 such that the desired value of the probability level is bracketed and then determine the percentage point by inverse interpolation.

The Gupta-Sobel table is for the percentage points of $\sqrt{2}\,Y$ and they use the Cornish-Fisher expansion with four adjustment terms.

Krishnaiah and Armitage [13] have tabulated the 95 and 99 percent points for $k = 1(1)10$, $\nu = 5(1)35$, and $\rho = 0(0.1)0.9$. They first evaluated $G_{k,\nu}(y;\ \rho)$ in (2.3) for $y = 0.1(0.20)6.1$ for the different values of ρ, ν, and k using the 40-point Gauss-Hermite quadrature formula for the inner integral and 48-point Gauss quadrature formula for the outer integral. The required percentage points were then obtained by using cubic interpolation.

5. CONSTRUCTION OF THE NEW TABLE

The upper 100α percent point of the distribution of Y is denoted by $y_\alpha^* \equiv y^*(\alpha,k,\nu,\rho)$. Its numerical value is obtained as the solution of

$$G_{\nu,m}(y) = 1-\alpha. \tag{5.1}$$

The secant method was used to find a value of y such that

$$|G_{\nu,m}(y)-1+\alpha| \leq \epsilon \tag{5.2}$$

with $\epsilon = 10^{-8}$. This value of y satisfying (5.2) is our solution y_α^*.

Now, the numerical evaluation of $G_{\nu,m}(y)$ involves the evaluation of the integrals $F^{(j)}(y)$ given by (3.9) for $j = 0,1,\ldots,8$. This was accomplished by using one of two different methods, labeled Method 1 and Method 2, depending on the value of ρ; the only exception is when $\nu = \infty$ in which case Method 2 was used for all the selected values of ρ. Method 1 evaluates $F^{(j)}(y)$ using the 60-point Gauss-Hermite quadrature formula. We will denote this numerically evaluated quantity by $\tilde{F}^{(j)}(y)$. To describe Method 2, letting

$$A = \rho^{\frac{1}{2}}(1-\rho)^{-\frac{1}{2}},\ B = (1-\rho)^{-\frac{1}{2}}, \tag{5.3}$$

and changing the variable by the transformation $u = At + By$, we can write (3.9) as

$$(5.4) \qquad F^{(j)}(y) = \int_{-\infty}^{\infty} \Phi^k(u) H_j\left(\frac{u-By}{A}\right)\varphi\left(\frac{u-By}{A}\right)\left(\frac{B}{A}\right)^j \frac{1}{A}\,du.$$

The value of this integral is approximated by

$$(5.5) \qquad \tilde{F}^{(j)}(y) = \int_{-9}^{9A+4B} \Phi^k(u) H_j\left(\frac{u-By}{A}\right)\varphi\left(\frac{u-By}{A}\right)\left(\frac{B}{A}\right)^j \frac{1}{A}\,du.$$

and the integration was carried out by Gauss's method over intervals of length $D = 0.5$ starting from -9 until $9A + 4B$ was included. Note that, for convenience, the approximation for $F^{(j)}(y)$ is denoted by $\tilde{F}^{(j)}(y)$ for either method. Whatever the method, $\tilde{F}^{(j)}(y)$ is evaluated as a sum of the form $\sum_{\ell} F_{\ell}^{(j)}(t_{\ell};\, y)$. While evaluating the terms of this sum, we used the IMSL subroutine for the values of the standard normal cdf. If the exponent in $e^{-t_{\ell}^2/2}$ was less than -50 [i.e., $|t_{\ell}| > 10$], the value of $F_{\ell}^{(j)}(t_{\ell};\, y)$ was taken to be zero because for the ranges of ρ and ν in our table, the value of the term was not more than 10^{-10}. Also, if the value of the standard normal cdf involved in the expression was less than 10^{-5}, then for $k \geq 8$, the value of $F_{\ell}^{(j)}(t_{\ell};\, y)$ was taken to be zero since the value was not more than 10^{-20} for all the ρ values of the table.

Now, Method 2 is more precise than Method 1; however, it is also more expensive, the cost ratio being 6 to 1 in some extreme cases. In order to decide on the method and the number of correction terms for the range of selected values of ρ and ν, some preliminary comparative studies were made.

In the case of $k = 19$ and $\alpha = 0.01$, we computed the percentage point y^* using both methods with $m = 4$ correction terms for $\nu = 15$ and 120. The results were as follows.

Table 1. Upper 1% point of the distribution of Y

$k = 19$, $m = 4$

ρ	ν	Method 1	Method 2
0.2	15	3.96044429	3.96044429
	120	3.34250119	3.34250119
0.3	15	3.90335614	3.90335614
	120	3.32690446	3.32690446
0.5	15	3.78170980	3.78170981
	120	3.26561715	3.26561715
0.6	15	3.70147425	3.70147458
	120	3.21375097	3.21375094
0.7	15	3.59638560	3.61431834
	120	3.14096412	3.14096740

Based on this comparison, Method 1 was used for $\rho = 0.2(0.1)0.6$, and Method 2 for $\rho = 0.7(0.1)0.9$. In the same case ($k = 19$, $\alpha = 0.01$), the following table gives the percentage point y^* for $\rho = 0.1$ using both methods with 4 correction terms.

Table 2. Upper 1% point of the distribution of Y

$k = 19$, $m = 4$, $\rho = 0.1$

ν	Method 1	Method 2
15	4.01116552	4.01119416
30	3.60648913	3.60658967
48	3.47577569	3.47577575
60	3.43358259	3.43358261
120	3.35129394	3.35129394

Based on this, it was decided to use, in the case of $\rho = 0.1$, Method 2 for $\nu \leq 24$ and Method 1 for $\nu \geq 30$.

To illustrate the difference between using three and four correction terms, we give below the upper 1 percent point in the case of $k = 17$ and $\rho = 0.7$ for $\nu = 48$, 60, and 120 using Method 2.

Table 3. Upper 1% point of the distribution of Y

$k = 17$, $\rho = 0.7$

ν	3 terms	4 terms
48	2.94535892	2.94537526
60	2.92086206	2.92086933
120	2.87288348	2.87288399

For a high value of ρ, three terms seem to be quite adequate for $\nu \geq 60$. For small values of ρ, we used three terms for $\nu = 120$ and four terms for other selected values of ρ which are all less than or equal to 60.

We summarize below the ranges of ρ, k, ν, and α for which the percentage points of Y are given in Table 4 along with information regarding the method (i.e.,Method 1 or 2) and the number of correction terms (m) used. In Table 4, it should be noted that $P^* = 1-\alpha$. The entries in the table are accurate to five decimal places.

The coverage of Table 4

(i) $\rho = 0.1$; $k = 1(1)9(2)19$; $\alpha = 1-P^* = 0.01,0.05,0.10,0.25$

ν	Method	m
15(1)20,24	2	4
30,36,48,60	1	4
120,∞	1	3

(ii) $\rho = 0.2(0.1)0.6$; $k = 1(1)9$; $\alpha = 1-P^* = 0.01,0.05,0.10,0.25$; Method 1
$\rho = 0.2(0.1)0.6$; $k = 11(2)19$; $\alpha = 1-P^* = 0.01,0.05,0.10,0.25$; Method 2

ν	m
15(1)20,24,30,36,48,60	4
120,∞	3

(iii) $\rho = 0.7(0.1)0.9$; $k = 1(1)9(2)15$; $\alpha = 1-P^* = 0.05,0.10$; Method 2

ν	m
15,17,20,24,36	4
60,120,∞	3

TABLE IV (cont.)

$\rho = .9$

ν	P*	K: 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.
15.	.90	1.34061	1.51207	1.60070	1.65918	1.70230	1.73620	1.76399	1.78745	1.80770	1.84129	1.86842	1.89110
	.95	1.75305	1.92577	2.01578	2.07542	2.11952	2.15425	2.18277	2.20687	2.22769	2.26227	2.29024	2.31365
17.	.90	1.33338	1.50318	1.59089	1.64875	1.69139	1.72492	1.75239	1.77558	1.79560	1.82880	1.85561	1.87803
	.95	1.73961	1.90979	1.99841	2.05711	2.10049	2.13466	2.16270	2.18641	2.20688	2.24087	2.26837	2.29137
20.	.90	1.32534	1.49330	1.58000	1.63716	1.67928	1.71239	1.73951	1.76241	1.78217	1.81494	1.84140	1.86352
	.95	1.72472	1.89211	1.97920	2.03687	2.07947	2.11301	2.14054	2.16380	2.18389	2.21725	2.24422	2.26678
24.	.90	1.31784	1.48409	1.56984	1.62636	1.66800	1.70071	1.72752	1.75014	1.76966	1.80203	1.82817	1.85001
	.95	1.71088	1.87570	1.96138	2.01809	2.05997	2.09294	2.11999	2.14284	2.16258	2.19534	2.22183	2.24399
36.	.90	1.30551	1.46898	1.55319	1.60866	1.64951	1.68159	1.70787	1.73005	1.74918	1.78090	1.80650	1.82790
	.95	1.68830	1.84894	1.93234	1.98750	2.02821	2.06025	2.08653	2.10873	2.12790	2.15971	2.18541	2.20691
60.	.90	1.29582	1.45710	1.54012	1.59477	1.63500	1.66659	1.69247	1.71430	1.73312	1.76433	1.78952	1.81056
	.95	1.67065	1.82806	1.90970	1.96365	2.00346	2.03478	2.06046	2.08215	2.10088	2.13195	2.15705	2.17804
120.	.90	1.28865	1.44832	1.53045	1.58450	1.62428	1.65551	1.68109	1.70266	1.72126	1.75210	1.77698	1.79777
	.95	1.65765	1.81270	1.89304	1.94611	1.98527	2.01606	2.04131	2.06263	2.08103	2.11155	2.13622	2.15683
∞	.90	1.28155	1.43965	1.52091	1.57437	1.61370	1.64458	1.66985	1.69118	1.70956	1.74003	1.76461	1.78514
	.95	1.64485	1.79759	1.87666	1.92888	1.96738	1.99766	2.02248	2.04344	2.06152	2.09152	2.11575	2.13600

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .8$

ν	P*	K: 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.
15.	.90	1.34061	1.57096	1.69174	1.77202	1.83151	1.87843	1.91700	1.94963	1.97784	2.02473	2.06269	2.09449
	.95	1.75305	1.98203	2.10348	2.18470	2.24511	2.29289	2.33225	2.36561	2.39450	2.44258	2.48159	2.51431
17.	.90	1.33338	1.56139	1.68082	1.76017	1.81894	1.86528	1.90337	1.93558	1.96343	2.00971	2.04718	2.07855
	.95	1.73961	1.96506	2.08449	2.16431	2.22365	2.27058	2.30923	2.34197	2.37032	2.41750	2.45577	2.48785
20.	.90	1.32534	1.55076	1.66870	1.74701	1.80498	1.85069	1.88824	1.92001	1.94746	1.99306	2.02998	2.06088
	.95	1.72472	1.94623	2.06350	2.14179	2.19996	2.24595	2.28381	2.31588	2.34364	2.38984	2.42729	2.45869
24.	.90	1.31784	1.54085	1.65740	1.73475	1.79199	1.83711	1.87417	1.90550	1.93258	1.97757	2.01397	2.04443
	.95	1.71088	1.92886	2.04404	2.12091	2.17801	2.22313	2.26027	2.29172	2.31894	2.36423	2.40093	2.43170
36.	.90	1.30551	1.52460	1.63890	1.71468	1.77073	1.81488	1.85113	1.88178	1.90825	1.95222	1.98778	2.01754
	.95	1.68830	1.90047	2.01234	2.08693	2.14228	2.18601	2.22197	2.25243	2.27877	2.32259	2.35809	2.38784
60.	.90	1.29582	1.51184	1.62439	1.69894	1.75406	1.79745	1.83308	1.86319	1.88919	1.93237	1.96728	1.99649
	.95	1.67065	1.87832	1.98764	2.06045	2.11446	2.15710	2.19217	2.22185	2.24752	2.29020	2.32476	2.35372
120.	.90	1.28865	1.50241	1.61366	1.68732	1.74175	1.78459	1.81976	1.84947	1.87513	1.91772	1.95215	1.98095
	.95	1.65765	1.86204	1.96949	2.04100	2.09403	2.13587	2.17028	2.19939	2.22457	2.26642	2.30030	2.32868
∞	.90	1.28155	1.49310	1.60308	1.67585	1.72960	1.77191	1.80662	1.83594	1.86126	1.90327	1.93724	1.96564
	.95	1.64485	1.84602	1.95164	2.02189	2.07395	2.11502	2.14878	2.17734	2.20203	2.24307	2.27628	2.30409

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .7$

ν	P*	K: 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.
15.	.90	1.34061	1.61114	1.75452	1.85041	1.92173	1.97816	2.02464	2.06404	2.09814	2.15496	2.20104	2.23969
	.95	1.75305	2.01913	2.16213	2.25846	2.33046	2.38762	2.43483	2.47494	2.50972	2.56779	2.61500	2.65467
17.	.90	1.33338	1.60105	1.74274	1.83742	1.90782	1.96349	2.00934	2.04820	2.08183	2.13782	2.18324	2.22133
	.95	1.73961	2.00141	2.14189	2.23645	2.30708	2.36313	2.40941	2.44871	2.48280	2.53966	2.58589	2.62472
20.	.90	1.32534	1.58985	1.72966	1.82301	1.89238	1.94723	1.99238	2.03063	2.06373	2.11884	2.16353	2.20099
	.95	1.72472	1.98182	2.11954	2.21215	2.28128	2.33612	2.38138	2.41981	2.45313	2.50870	2.55386	2.59178
24.	.90	1.31784	1.57941	1.71747	1.80959	1.87801	1.93208	1.97659	2.01428	2.04690	2.10118	2.14519	2.18207
	.95	1.71088	1.96365	2.09881	2.18963	2.25739	2.31111	2.35543	2.39306	2.42567	2.48004	2.52421	2.56130
36.	.90	1.30551	1.56230	1.69752	1.78763	1.85450	1.90731	1.95076	1.98754	2.01936	2.07228	2.11516	2.15108
	.95	1.68830	1.93404	2.06509	2.15301	2.21854	2.27045	2.31326	2.34958	2.38105	2.43349	2.47606	2.51179
60.	.90	1.29582	1.54887	1.68187	1.77042	1.83608	1.88792	1.93054	1.96661	1.99781	2.04968	2.09169	2.12688
	.95	1.67065	1.91096	2.03882	2.12449	2.18830	2.23882	2.28046	2.31577	2.34636	2.39731	2.43865	2.47334
120.	.90	1.28665	1.53894	1.67032	1.75772	1.82249	1.87360	1.91562	1.95117	1.98192	2.03301	2.07438	2.10902
	.95	1.65765	1.89399	2.01952	2.10355	2.16610	2.21560	2.25638	2.29096	2.32090	2.37076	2.41121	2.44513
∞	.90	1.28155	1.52914	1.65892	1.74518	1.80909	1.85949	1.90091	1.93595	1.96624	2.01658	2.05732	2.09143
	.95	1.64485	1.87730	2.00055	2.08298	2.14429	2.19280	2.23274	2.26660	2.29591	2.34470	2.38427	2.41744

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

ρ = .6 (cont.)

ν	P*	K 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
30.	.75	.68276	.99879	1.16244	1.27058	1.35038	1.41317	1.46466	1.50815	1.54570	1.60801	1.65836	1.70046	1.73655	1.76806
	.90	1.31042	1.59767	1.75022	1.85239	1.92847	1.98870	2.03835	2.08046	2.11692	2.17768	2.22699	2.26835	2.30391	2.33503
	.95	1.69726	1.97043	2.11757	2.21688	2.29121	2.35028	2.39910	2.44061	2.47663	2.53677	2.58571	2.62685	2.66226	2.69330
	.99	2.45726	2.71007	2.84954	2.94491	3.01692	3.07452	3.12237	3.16322	3.19879	3.25841	3.30715	3.34826	3.38375	3.41493
36.	.75	.68137	.99638	1.15940	1.26708	1.34653	1.40902	1.46026	1.50354	1.54090	1.60289	1.65297	1.69485	1.73074	1.76207
	.90	1.30551	1.59058	1.74180	1.84301	1.91834	1.97797	2.02710	2.06876	2.10484	2.16493	2.21369	2.25460	2.28974	2.32050
	.95	1.68830	1.95829	2.10353	2.20148	2.27474	2.33295	2.38105	2.42193	2.45740	2.51661	2.56477	2.60525	2.64009	2.67062
	.99	2.43449	2.68174	2.81791	2.91095	2.98115	3.03729	3.08392	3.12371	3.15835	3.21641	3.26385	3.30387	3.33841	3.36875
48.	.75	.67964	.99338	1.15562	1.26273	1.34174	1.40386	1.45480	1.49782	1.53494	1.59653	1.64629	1.68788	1.72352	1.75464
	.90	1.29944	1.58181	1.73138	1.83141	1.90582	1.96469	2.01320	2.05431	2.08991	2.14918	2.19726	2.23758	2.27223	2.30254
	.95	1.67722	1.94332	2.08621	2.18248	2.25445	2.31159	2.35880	2.39891	2.43370	2.49176	2.53898	2.57865	2.61278	2.64269
	.99	2.40658	2.64705	2.77920	2.86935	2.93739	2.99174	3.03687	3.07536	3.10886	3.16500	3.21085	3.24951	3.28288	3.31219
60.	.75	.67860	.99160	1.15336	1.26014	1.33888	1.40079	1.45155	1.49440	1.53139	1.59274	1.64230	1.68373	1.71922	1.75021
	.90	1.29582	1.57659	1.72518	1.82451	1.89838	1.95681	2.00493	2.04572	2.08103	2.13982	2.18750	2.22748	2.26182	2.29187
	.95	1.67065	1.93444	2.07594	2.17122	2.24242	2.29894	2.34562	2.38527	2.41966	2.47704	2.52369	2.56289	2.59660	2.62614
	.99	2.39012	2.62661	2.75640	2.84492	2.91164	2.96494	3.00917	3.04690	3.07974	3.13474	3.17966	3.21752	3.25020	3.27889
120.	.75	.67654	.98804	1.14888	1.25498	1.33320	1.39468	1.44508	1.48762	1.52433	1.58521	1.63439	1.67548	1.71069	1.74142
	.90	1.28865	1.56625	1.71291	1.81086	1.88365	1.94120	1.98858	2.02872	2.06347	2.12130	2.16818	2.20748	2.24123	2.27076
	.95	1.65765	1.91691	2.05567	2.14900	2.21869	2.27398	2.31962	2.35837	2.39198	2.44802	2.49356	2.53181	2.56470	2.59352
	.99	2.35782	2.58658	2.71177	2.79703	2.86123	2.91248	2.95499	2.99123	3.02276	3.07554	3.11863	3.15494	3.18626	3.21375
∞	.75	.67449	.98450	1.14442	1.24987	1.32757	1.38863	1.43867	1.48090	1.51734	1.57776	1.62655	1.66731	1.70223	1.73271
	.90	1.28155	1.55604	1.70081	1.79739	1.86912	1.92581	1.97246	2.01197	2.04616	2.10304	2.14914	2.18777	2.22094	2.24995
	.95	1.64485	1.89967	2.03577	2.12719	2.19540	2.24948	2.29410	2.33198	2.36480	2.41953	2.46399	2.50131	2.53340	2.56150
	.99	2.32635	2.54762	2.66839	2.75050	2.81226	2.86152	2.90236	2.93716	2.96742	3.01806	3.05938	3.09417	3.12417	3.15050

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .6$

ν	P*	K 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
15.	.75	.69120	1.01348	1.18103	1.29199	1.37400	1.43859	1.49161	1.53642	1.57513	1.63941	1.69140	1.73490	1.77220	1.80479
	.90	1.34061	1.64150	1.80239	1.91055	1.99128	2.05532	2.10818	2.15306	2.19197	2.25686	2.30960	2.35390	2.39200	2.42538
	.95	1.75305	2.04627	2.20553	2.31330	2.39454	2.45909	2.51255	2.55805	2.59759	2.66370	2.71759	2.76295	2.80204	2.83634
	.99	2.60240	2.89103	3.05135	3.16122	3.24423	3.31061	3.36574	3.41276	3.45368	3.52220	3.57812	3.62523	3.66584	3.70147
16.	.75	.69013	1.01162	1.17868	1.28927	1.37100	1.43536	1.48819	1.53283	1.57140	1.63542	1.68721	1.73053	1.76767	1.80012
	.90	1.33676	1.63589	1.79571	1.90310	1.98323	2.04678	2.09922	2.14374	2.18234	2.24670	2.29899	2.34291	2.38068	2.41376
	.95	1.74589	2.03650	2.19417	2.30101	2.38117	2.44500	2.49785	2.54282	2.58190	2.64722	2.70045	2.74525	2.78385	2.81772
	.99	2.58344	2.86742	3.02510	3.13320	3.21488	3.28024	3.33454	3.38087	3.42120	3.48877	3.54393	3.59042	3.63052	3.66571
17.	.75	.68920	1.00999	1.17661	1.28689	1.36837	1.43253	1.48518	1.52968	1.56811	1.63192	1.68352	1.72668	1.76369	1.79602
	.90	1.33338	1.63098	1.78986	1.89657	1.97618	2.03929	2.09138	2.13558	2.17390	2.23779	2.28970	2.33328	2.37077	2.40359
	.95	1.73961	2.02794	2.18424	2.29009	2.36949	2.43269	2.48501	2.52953	2.56820	2.63284	2.68550	2.72981	2.76799	2.80148
	.99	2.56690	2.84680	3.00215	3.10664	3.18914	3.25356	3.30709	3.35278	3.39256	3.45922	3.51367	3.55958	3.59918	3.63395
18.	.75	.68836	1.00854	1.17477	1.28477	1.36604	1.43001	1.48252	1.52688	1.56520	1.62881	1.68025	1.72328	1.76017	1.79239
	.90	1.33039	1.62663	1.78468	1.89080	1.96994	2.03268	2.08444	2.12838	2.16645	2.22993	2.28150	2.32479	2.36202	2.39462
	.95	1.73407	2.02039	2.17548	2.28047	2.35919	2.42185	2.47370	2.51782	2.55613	2.62017	2.67234	2.71623	2.75403	2.78719
	.99	2.55236	2.82866	2.98193	3.08699	3.16641	3.22997	3.28280	3.32790	3.36717	3.43299	3.48677	3.53212	3.57125	3.60562
19.	.75	.68762	1.00725	1.17313	1.28289	1.36396	1.42777	1.48014	1.52439	1.56261	1.62605	1.67734	1.72024	1.75703	1.78915
	.90	1.32773	1.62276	1.78008	1.88566	1.96439	2.02680	2.07828	2.12196	2.15982	2.22293	2.27420	2.31723	2.35423	2.38663
	.95	1.72913	2.01369	2.16770	2.27192	2.35004	2.41221	2.46366	2.50742	2.54542	2.60893	2.66066	2.70417	2.74165	2.77452
	.99	2.53947	2.81258	2.96399	3.06776	3.14620	3.20899	3.26118	3.30573	3.34453	3.40958	3.46273	3.50757	3.54626	3.58024
20.	.75	.68695	1.00609	1.17166	1.28119	1.36209	1.42576	1.47801	1.52216	1.56028	1.62356	1.67473	1.71752	1.75421	1.78625
	.90	1.32534	1.61930	1.77595	1.88106	1.95942	2.02153	2.07275	2.11622	2.15389	2.21667	2.26766	2.31046	2.34727	2.37949
	.95	1.72472	2.00768	2.16074	2.26427	2.34187	2.40360	2.45468	2.49812	2.53585	2.59889	2.65022	2.69340	2.73059	2.76320
	.99	2.52797	2.79822	2.94797	3.05058	3.12815	3.19023	3.24183	3.28588	3.32425	3.38858	3.44116	3.48551	3.52379	3.55742
24.	.75	.68485	1.00242	1.16703	1.27586	1.35621	1.41944	1.47131	1.51512	1.55296	1.61575	1.66651	1.70895	1.74533	1.77711
	.90	1.31784	1.60841	1.76300	1.86663	1.94384	2.00500	2.05543	2.09821	2.13527	2.19703	2.24717	2.28925	2.32542	2.35709
	.95	1.71088	1.98889	2.13896	2.24035	2.31630	2.37668	2.42662	2.46908	2.50594	2.56752	2.61764	2.65978	2.69607	2.72788
	.99	2.49216	2.75355	2.89809	2.99705	3.07182	3.13165	3.18138	3.22383	3.26080	3.32280	3.37348	3.41623	3.45315	3.48558

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

ρ = .5 (cont.)

ν	P*	K: 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
30.	.75	.68276	1.02892	1.20887	1.32807	1.41620	1.48562	1.54261	1.59079	1.63241	1.70154	1.75745	1.80423	1.84435	1.87940
	.90	1.31042	1.62007	1.78552	1.89676	1.97982	2.04571	2.10012	2.14633	2.18639	2.25323	2.30756	2.35320	2.39246	2.42686
	.95	1.69726	1.98903	2.14730	2.25461	2.33519	2.39938	2.45256	2.49783	2.53718	2.60299	2.65664	2.70181	2.74075	2.77491
	.99	2.45726	2.72303	2.87084	2.97247	3.04950	3.11131	3.16279	3.20682	3.24523	3.30976	3.36262	3.40730	3.44594	3.47993
36.	.75	.68137	1.02638	1.20560	1.32426	1.41196	1.48103	1.53772	1.58564	1.62704	1.69577	1.75136	1.79786	1.83774	1.87258
	.90	1.30551	1.61274	1.77666	1.88678	1.96896	2.03414	2.08794	2.13362	2.17322	2.23927	2.29294	2.33802	2.37679	2.41074
	.95	1.68830	1.97656	2.13267	2.23841	2.31776	2.38095	2.43327	2.47781	2.51651	2.58121	2.63395	2.67833	2.71658	2.75013
	.99	2.43449	2.69421	2.83835	2.93735	3.01235	3.07249	3.12256	3.16538	3.20272	3.26545	3.31682	3.36023	3.39777	3.43078
48.	.75	.67964	1.02322	1.20153	1.31952	1.40670	1.47533	1.53165	1.57925	1.62036	1.68861	1.74380	1.78996	1.82953	1.86410
	.90	1.29944	1.60365	1.76569	1.87444	1.95554	2.01984	2.07289	2.11791	2.15694	2.22200	2.27486	2.31924	2.35740	2.39082
	.95	1.67722	1.96117	2.11462	2.21844	2.29628	2.35824	2.40951	2.45314	2.49104	2.55439	2.60599	2.64941	2.68682	2.71962
	.99	2.40658	2.65892	2.79860	2.89439	2.96689	3.02499	3.07334	3.11467	3.15069	3.21119	3.26072	3.30256	3.33872	3.37052
60.	.75	.67860	1.02134	1.19910	1.31670	1.40356	1.47193	1.52803	1.57544	1.61638	1.68434	1.73929	1.78525	1.82464	1.85905
	.90	1.29582	1.59825	1.75916	1.86710	1.94757	2.01134	2.06394	2.10858	2.14726	2.21174	2.26412	2.30808	2.34588	2.37898
	.95	1.67065	1.95205	2.10392	2.20660	2.28355	2.34478	2.39544	2.43853	2.47596	2.53850	2.58943	2.63228	2.66918	2.70155
	.99	2.39012	2.63813	2.77519	2.86911	2.94014	2.99704	3.04438	3.08482	3.12008	3.17926	3.22769	3.26860	3.30395	3.33504
120.	.75	.67654	1.01759	1.19428	1.31108	1.39732	1.46518	1.52085	1.56788	1.60848	1.67587	1.73034	1.77589	1.81493	1.84902
	.90	1.28865	1.58755	1.74625	1.85258	1.93178	1.99451	2.04624	2.09011	2.12812	2.19145	2.24286	2.28601	2.32309	2.35556
	.95	1.65765	1.93403	2.08281	2.18325	2.25845	2.31824	2.36768	2.40972	2.44621	2.50716	2.55677	2.59850	2.63442	2.66591
	.99	2.35782	2.59742	2.72938	2.81963	2.88780	2.94236	2.98772	3.02645	3.06019	3.11680	3.16310	3.20218	3.23594	3.26562
∞	.75	.67449	1.01387	1.18949	1.30552	1.39114	1.45849	1.51373	1.56038	1.60065	1.66748	1.72148	1.76662	1.80531	1.83909
	.90	1.28155	1.57699	1.73352	1.83827	1.91623	1.97793	2.02879	2.07191	2.10925	2.17144	2.22191	2.26425	2.30063	2.33246
	.95	1.64485	1.91633	2.06208	2.16033	2.23382	2.29219	2.34044	2.38144	2.41702	2.47641	2.52473	2.56534	2.60029	2.63093
	.99	2.32635	2.55782	2.68485	2.77156	2.83698	2.88927	2.93271	2.96978	3.00206	3.05618	3.10040	3.13772	3.16993	3.19823

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .5$

ν	P*	K 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
15.	.75	.69120	1.04441	1.22889	1.35143	1.44218	1.51376	1.57259	1.62236	1.66539	1.73691	1.79482	1.84330	1.88491	1.92127
	.90	1.34061	1.66548	1.84048	1.95867	2.04719	2.11757	2.17579	2.22529	2.26827	2.34008	2.39855	2.44773	2.49009	2.52723
	.95	1.75305	2.06701	2.23903	2.35629	2.44467	2.51527	2.57388	2.62387	2.66738	2.74028	2.79984	2.85006	2.89340	2.93148
	.99	2.60240	2.90713	3.07800	3.19577	3.28507	3.35669	3.41629	3.46722	3.51161	3.58607	3.64696	3.69835	3.74272	3.78171
16.	.75	.69013	1.04245	1.22635	1.34846	1.43888	1.51018	1.56878	1.61835	1.66120	1.73242	1.79007	1.83834	1.87975	1.91595
	.90	1.33676	1.65967	1.83344	1.95074	2.03855	2.10835	2.16607	2.21515	2.25775	2.32891	2.38684	2.43556	2.47751	2.51429
	.95	1.74589	2.05696	2.22718	2.34314	2.43049	2.50025	2.55814	2.60750	2.65046	2.72243	2.78120	2.83075	2.87351	2.91106
	.99	2.58344	2.88312	3.05110	3.16691	3.25477	3.32528	3.38398	3.43415	3.47789	3.55130	3.61135	3.66204	3.70582	3.74429
17.	.75	.68920	1.04072	1.22412	1.34586	1.43598	1.50704	1.56544	1.61483	1.65752	1.72847	1.78590	1.83398	1.87523	1.91128
	.90	1.33338	1.65458	1.82727	1.94378	2.03098	2.10027	2.15757	2.20627	2.24854	2.31914	2.37660	2.42491	2.46651	2.50298
	.95	1.73961	2.04816	2.21682	2.33164	2.41810	2.48713	2.54440	2.59322	2.63570	2.70685	2.76494	2.81391	2.85616	2.89327
	.99	2.56690	2.86215	3.02756	3.14161	3.22816	3.29763	3.35549	3.40496	3.44810	3.52052	3.57980	3.62984	3.67307	3.71106
18.	.75	.68836	1.03919	1.22215	1.34355	1.43342	1.50426	1.56247	1.61171	1.65426	1.72497	1.78220	1.83012	1.87122	1.90715
	.90	1.33039	1.65008	1.82182	1.93764	2.02429	2.09314	2.15005	2.19843	2.24041	2.31051	2.36756	2.41552	2.45681	2.49300
	.95	1.73407	2.04040	2.20768	2.32151	2.40719	2.47557	2.53229	2.58064	2.62271	2.69314	2.75064	2.79910	2.84091	2.87761
	.99	2.55236	2.84369	3.00681	3.11927	3.20463	3.27315	3.33024	3.37906	3.42164	3.49314	3.55168	3.60112	3.64384	3.68139
19.	.75	.68762	1.03783	1.22038	1.34150	1.43113	1.50178	1.55983	1.60893	1.65136	1.72186	1.77891	1.82667	1.86765	1.90346
	.90	1.32773	1.64607	1.81697	1.93217	2.01834	2.08679	2.14337	2.19145	2.23318	2.30284	2.35952	2.40717	2.44818	2.48413
	.95	1.72913	2.03350	2.19956	2.31251	2.39749	2.46531	2.52155	2.56948	2.61118	2.68098	2.73795	2.78596	2.82738	2.86374
	.99	2.53947	2.82733	2.98839	3.09942	3.18370	3.25136	3.30773	3.35595	3.39801	3.46866	3.52652	3.57539	3.61762	3.65476
20.	.75	.68695	1.03661	1.21880	1.33965	1.42907	1.49956	1.55746	1.60643	1.64875	1.71906	1.77596	1.82359	1.86444	1.90015
	.90	1.32534	1.64248	1.81262	1.92727	2.01301	2.08111	2.13738	2.18521	2.22670	2.29597	2.35233	2.39969	2.44046	2.47620
	.95	1.72472	2.02733	2.19230	2.30446	2.38883	2.45614	2.51195	2.55951	2.60087	2.67012	2.72662	2.77423	2.81530	2.85135
	.99	2.52797	2.81273	2.97195	3.08169	3.16498	3.23185	3.28757	3.33524	3.37682	3.44668	3.50389	3.55223	3.59402	3.63076
24.	.75	.68485	1.03275	1.21382	1.33384	1.42261	1.49256	1.55000	1.59857	1.64054	1.71026	1.76666	1.81386	1.85435	1.88973
	.90	1.31784	1.63120	1.79898	1.91191	1.99629	2.06328	2.11861	2.16562	2.20640	2.27444	2.32977	2.37627	2.41628	2.45134
	.95	1.71088	2.00801	2.16959	2.27930	2.36175	2.42748	2.48196	2.52836	2.56870	2.63620	2.69126	2.73764	2.77762	2.81271
	.99	2.49216	2.76727	2.92072	3.02637	3.10652	3.17086	3.22447	3.27034	3.31035	3.37758	3.43267	3.47923	3.51949	3.55490

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .4$ (cont.)

ν	P*	K 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
30.	.75	.68276	1.05449	1.24834	1.37703	1.47233	1.54749	1.60926	1.66152	1.70671	1.78180	1.84261	1.89352	1.93721	1.97540
	.90	1.31042	1.63802	1.81388	1.93250	2.02130	2.09189	2.15026	2.19990	2.24299	2.31498	2.37359	2.42288	2.46533	2.50255
	.95	1.69726	2.00332	2.17025	2.28385	2.36940	2.43771	2.49440	2.54275	2.58482	2.65530	2.71287	2.76142	2.80331	2.84011
	.99	2.45726	2.73215	2.88594	2.99212	3.07287	3.13782	3.19204	3.23849	3.27908	3.34740	3.40350	3.45099	3.49212	3.52835
36.	.75	.68137	1.05184	1.24485	1.37291	1.46771	1.54246	1.60388	1.65583	1.70074	1.77537	1.83579	1.88637	1.92977	1.96770
	.90	1.30551	1.63046	1.80460	1.92196	2.00975	2.07951	2.13718	2.18621	2.22875	2.29981	2.35764	2.40627	2.44814	2.48484
	.95	1.68830	1.99056	2.15508	2.26693	2.35108	2.41825	2.47397	2.52146	2.56278	2.63199	2.68849	2.73611	2.77720	2.81329
	.99	2.43449	2.70292	2.85273	2.95603	3.03452	3.09762	3.15027	3.19537	3.23476	3.30105	3.35547	3.40153	3.44142	3.47655
48.	.75	.67964	1.04853	1.24050	1.36779	1.46198	1.53621	1.59719	1.64877	1.69334	1.76739	1.82732	1.87749	1.92053	1.95813
	.90	1.29944	1.62109	1.79312	1.90892	1.99548	2.06422	2.12101	2.16928	2.21115	2.28106	2.33793	2.38573	2.42688	2.46294
	.95	1.67722	1.97481	2.13638	2.24607	2.32852	2.39428	2.44879	2.49524	2.53564	2.60327	2.65845	2.70495	2.74505	2.78026
	.99	2.40658	2.66714	2.81210	2.91188	2.98761	3.04844	3.09917	3.14259	3.18050	3.24428	3.29660	3.34087	3.37920	3.41294
60.	.75	.67860	1.04656	1.23791	1.36474	1.45856	1.53249	1.59321	1.64456	1.68893	1.76263	1.82228	1.87220	1.91501	1.95243
	.90	1.29582	1.61553	1.78630	1.90118	1.98700	2.05513	2.11141	2.15922	2.20069	2.26992	2.32622	2.37353	2.41425	2.44993
	.95	1.67065	1.96547	2.12530	2.23371	2.31515	2.38007	2.43388	2.47971	2.51956	2.58625	2.64066	2.68649	2.72601	2.76069
	.99	2.39012	2.64607	2.78819	2.88591	2.96001	3.01951	3.06910	3.11153	3.14857	3.21086	3.26194	3.30515	3.34255	3.37547
120.	.75	.67654	1.04264	1.23276	1.35868	1.45176	1.52509	1.58529	1.63619	1.68016	1.75318	1.81225	1.86168	1.90407	1.94110
	.90	1.28865	1.60450	1.77280	1.88584	1.97022	2.03714	2.09240	2.13932	2.18000	2.24787	2.30304	2.34938	2.38925	2.42417
	.95	1.65765	1.94704	2.10344	2.20933	2.28878	2.35207	2.40447	2.44909	2.48786	2.55271	2.60558	2.65008	2.68845	2.72211
	.99	2.35782	2.60481	2.74139	2.83509	2.90603	2.96292	3.01029	3.05080	3.08614	3.14551	3.19416	3.23529	3.27086	3.30216
∞	.75	.67449	1.03874	1.22766	1.35267	1.44503	1.51776	1.57745	1.62790	1.67148	1.74382	1.80232	1.85126	1.89322	1.92987
	.90	1.28155	1.59362	1.75948	1.87073	1.95367	2.01942	2.07366	2.11970	2.15960	2.22613	2.28018	2.32556	2.36459	2.39876
	.95	1.64485	1.92893	2.08197	2.18540	2.26291	2.32458	2.37562	2.41904	2.45675	2.51979	2.57114	2.61435	2.65158	2.68422
	.99	2.32635	2.56468	2.69593	2.78575	2.85364	2.90801	2.95323	2.99188	3.02556	3.08211	3.12840	3.16750	3.20130	3.23103

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

ρ = .4

ν	P* \ K	1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
15.	.75	.69120	1.07072	1.26973	1.40227	1.50063	1.57834	1.64227	1.69642	1.74328	1.82123	1.88440	1.93735	1.98281	2.02257
	.90	1.34061	1.68483	1.87140	1.99794	2.09300	2.16877	2.23157	2.28506	2.33157	2.40942	2.47293	2.52645	2.57262	2.61315
	.95	1.75305	2.08318	2.26535	2.39015	2.48455	2.56017	2.62310	2.67687	2.72374	2.80243	2.86684	2.92124	2.96824	3.00956
	.99	2.60240	2.91883	3.09756	3.22133	3.31552	3.39127	3.45447	3.50859	3.55585	3.63537	3.70064	3.75591	3.80382	3.84607
16.	.75	.69013	1.06867	1.26702	1.39907	1.49704	1.57442	1.63808	1.69199	1.73863	1.81622	1.87910	1.93179	1.97703	2.01659
	.90	1.33676	1.67885	1.86403	1.98955	2.08380	2.15890	2.22112	2.27411	2.32018	2.39725	2.46012	2.51307	2.55874	2.59882
	.95	1.74589	2.07288	2.25306	2.37639	2.46962	2.54429	2.60639	2.65944	2.70568	2.78329	2.84681	2.90044	2.94679	2.98753
	.99	2.58344	2.89449	3.07011	3.19177	3.28438	3.35890	3.42109	3.47434	3.52085	3.59909	3.66328	3.71761	3.76465	3.80609
17.	.75	.68920	1.06686	1.26463	1.39625	1.49388	1.57098	1.63440	1.68810	1.73455	1.81182	1.87444	1.92690	1.97194	2.01133
	.90	1.33338	1.67359	1.85757	1.98220	2.07574	2.15025	2.21197	2.26453	2.31020	2.38661	2.44891	2.50138	2.54662	2.58632
	.95	1.73961	2.06387	2.24231	2.36437	2.45659	2.53041	2.59180	2.64423	2.68992	2.76659	2.82933	2.88229	2.92806	2.96829
	.99	2.56690	2.87323	3.04608	3.16583	3.25701	3.33040	3.39166	3.44413	3.48996	3.56706	3.63032	3.68384	3.73016	3.77096
18.	.75	.68836	1.06526	1.26252	1.39376	1.49108	1.56793	1.63113	1.68465	1.73094	1.80793	1.87031	1.92257	1.96744	2.00667
	.90	1.33039	1.66895	1.85187	1.97570	2.06862	2.14262	2.20390	2.25607	2.30140	2.37722	2.43904	2.49108	2.53594	2.57531
	.95	1.73407	2.05592	2.23284	2.35377	2.44510	2.51819	2.57895	2.63084	2.67605	2.75190	2.81394	2.86632	2.91157	2.95135
	.99	2.55236	2.85452	3.02489	3.14291	3.23280	3.30515	3.36556	3.41731	3.46252	3.53859	3.60100	3.65381	3.69952	3.73976
19.	.75	.68762	1.06383	1.26064	1.39154	1.48859	1.56521	1.62823	1.68157	1.72772	1.80445	1.86662	1.91871	1.96342	2.00252
	.90	1.32773	1.66482	1.84679	1.96992	2.06228	2.13582	2.19671	2.24854	2.29357	2.36888	2.43026	2.48193	2.52646	2.56554
	.95	1.72913	2.04865	2.22442	2.34436	2.43491	2.50735	2.56756	2.61896	2.66374	2.73886	2.80029	2.85215	2.89694	2.93632
	.99	2.53947	2.83792	3.00608	3.12254	3.21125	3.28266	3.34228	3.39337	3.43801	3.51313	3.57477	3.62693	3.67208	3.71183
20.	.75	.68695	1.06255	1.25895	1.38954	1.48635	1.56277	1.62562	1.67881	1.72482	1.80134	1.86332	1.91524	1.95982	1.99879
	.90	1.32534	1.66112	1.84223	1.96474	2.05661	2.12974	2.19028	2.24181	2.28657	2.36141	2.42240	2.47374	2.51798	2.55680
	.95	1.72472	2.04253	2.21689	2.33594	2.42579	2.49766	2.55737	2.60835	2.65275	2.72722	2.78811	2.83950	2.88389	2.92290
	.99	2.52797	2.82311	2.98928	3.10433	3.19196	3.26251	3.32142	3.37190	3.41601	3.49025	3.55118	3.60275	3.64739	3.68669
24.	.75	.68485	1.05851	1.25362	1.38326	1.47931	1.55509	1.61740	1.67013	1.71572	1.79152	1.85292	1.90434	1.94846	1.98705
	.90	1.31784	1.64949	1.82796	1.94851	2.03882	2.11067	2.17012	2.22069	2.26461	2.33801	2.39780	2.44811	2.49144	2.52945
	.95	1.71088	2.02275	2.19335	2.30965	2.39732	2.46738	2.52556	2.57521	2.61843	2.69088	2.75008	2.80003	2.84315	2.88104
	.99	2.49216	2.77702	2.93692	3.04751	3.13170	3.19946	3.25604	3.30453	3.34691	3.41825	3.47682	3.52642	3.56936	3.60718

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

ρ = .3 (cont.)

ν	P*	K 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
30.	.75	.68276	1.07648	1.28225	1.41912	1.52063	1.60079	1.66673	1.72258	1.77089	1.85127	1.91641	1.97100	2.01788	2.05888
	.90	1.31042	1.65251	1.83676	1.96140	2.05489	2.12935	2.19102	2.24353	2.28916	2.36550	2.42775	2.48017	2.52536	2.56503
	.95	1.69726	2.01436	2.18797	2.30648	2.39594	2.46753	2.52703	2.57785	2.62214	2.69644	2.75725	2.80859	2.85296	2.89198
	.99	2.45726	2.73853	2.89653	3.00596	3.08938	3.15664	3.21287	3.26113	3.30335	3.37454	3.43311	3.48278	3.52585	3.56384
36.	.75	.68137	1.07371	1.27854	1.41469	1.51562	1.59530	1.66084	1.71632	1.76431	1.84413	1.90881	1.96301	2.00954	2.05023
	.90	1.30551	1.64474	1.82713	1.95032	2.04268	2.11620	2.17706	2.22886	2.27386	2.34912	2.41047	2.46211	2.50662	2.54568
	.95	1.68830	2.00133	2.17232	2.28889	2.37681	2.44710	2.50551	2.55537	2.59879	2.67164	2.73121	2.78150	2.82495	2.86314
	.99	2.43449	2.70898	2.86272	2.96905	3.05002	3.11526	3.16978	3.21654	3.25744	3.32639	3.38309	3.43117	3.47285	3.50961
48.	.75	.67964	1.07027	1.27393	1.40920	1.50941	1.58849	1.65351	1.70855	1.75614	1.83527	1.89938	1.95308	1.99918	2.03948
	.90	1.29944	1.63512	1.81515	1.93662	2.02758	2.09993	2.15979	2.21072	2.25495	2.32887	2.38910	2.43978	2.48344	2.52175
	.95	1.67722	1.98527	2.15303	2.26722	2.35323	2.42194	2.47899	2.52767	2.57005	2.64108	2.69915	2.74814	2.79044	2.82762
	.99	2.40658	2.67280	2.82137	2.92390	3.00188	3.06463	3.11704	3.16196	3.20123	3.26738	3.32176	3.36784	3.40777	3.44298
60.	.75	.67860	1.06821	1.27118	1.40592	1.50571	1.58443	1.64915	1.70391	1.75127	1.82999	1.89375	1.94716	1.99299	2.03306
	.90	1.29582	1.62940	1.80805	1.92848	2.01861	2.09027	2.14954	2.19994	2.24371	2.31684	2.37640	2.42650	2.46967	2.50752
	.95	1.67065	1.97574	2.14161	2.25437	2.33926	2.40704	2.46329	2.51126	2.55302	2.62299	2.68016	2.72838	2.77000	2.80657
	.99	2.39012	2.65150	2.79704	2.89735	2.97356	3.03486	3.08601	3.12985	3.16815	3.23266	3.28565	3.33054	3.36943	3.40372
120.	.75	.67654	1.06412	1.26571	1.39940	1.49835	1.57637	1.64048	1.69471	1.74159	1.81950	1.88257	1.93538	1.98070	2.02031
	.90	1.28865	1.61808	1.79399	1.91237	2.00086	2.07115	2.12924	2.17861	2.22146	2.29302	2.35125	2.40022	2.44238	2.47934
	.95	1.65765	1.95694	2.11906	2.22905	2.31172	2.37765	2.43233	2.47892	2.51944	2.58731	2.64270	2.68940	2.72968	2.76506
	.99	2.35782	2.60979	2.74943	2.84541	2.91820	2.97665	3.02537	3.06709	3.10351	3.16477	3.21505	3.25761	3.29445	3.32690
∞	.75	.67449	1.06007	1.26029	1.39295	1.49106	1.56838	1.63188	1.68558	1.73199	1.80909	1.87148	1.92371	1.96851	2.00765
	.90	1.28155	1.60691	1.78012	1.89649	1.98336	2.05230	2.10923	2.15758	2.19953	2.26953	2.32645	2.37429	2.41546	2.45153
	.95	1.64485	1.93847	2.09693	2.20420	2.28470	2.34882	2.40194	2.44718	2.48650	2.55229	2.60594	2.65113	2.69009	2.72429
	.99	2.32635	2.56924	2.70321	2.79501	2.86449	2.92020	2.96658	3.00624	3.04084	3.09897	3.14662	3.18691	3.22176	3.25243

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .3$

ν	P*	K: 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
15.	.75	.69120	1.09341	1.30498	1.44625	1.55129	1.63440	1.70288	1.76093	1.81121	1.89494	1.96288	2.01987	2.06884	2.11169
	.90	1.34061	1.70061	1.89670	2.03020	2.13079	2.21116	2.27791	2.33488	2.38448	2.46767	2.53571	2.59316	2.64281	2.68648
	.95	1.75305	2.09588	2.28614	2.41703	2.51636	2.59613	2.66264	2.71956	2.76924	2.85276	2.92120	2.97904	3.02902	3.07295
	.99	2.60240	2.92734	3.11193	3.24032	3.33839	3.41757	3.48385	3.54083	3.59076	3.67524	3.74513	3.80480	3.85695	3.90336
16.	.75	.69013	1.09126	1.30210	1.44280	1.54740	1.63013	1.69829	1.75606	1.80609	1.88940	1.95699	2.01368	2.06239	2.10502
	.90	1.33676	1.69446	1.88902	2.02137	2.12104	2.20065	2.26673	2.32311	2.37219	2.45446	2.52172	2.57849	2.62752	2.67062
	.95	1.74589	2.08536	2.27344	2.40271	2.50074	2.57945	2.64504	2.70117	2.75016	2.83250	2.90000	2.95705	3.00638	3.04976
	.99	2.58344	2.90273	3.08400	3.21009	3.30642	3.38417	3.44923	3.50512	3.55406	3.63674	3.70495	3.76301	3.81357	3.85839
17.	.75	.68920	1.08938	1.29956	1.43978	1.54398	1.62638	1.69426	1.75179	1.80159	1.88453	1.95181	2.00824	2.05672	2.09914
	.90	1.33338	1.68906	1.88288	2.01364	2.11250	2.19144	2.25695	2.31282	2.36144	2.44293	2.50952	2.56569	2.61420	2.65683
	.95	1.73961	2.07616	2.26234	2.39020	2.48711	2.56488	2.62967	2.68511	2.73348	2.81479	2.88144	2.93778	2.98650	3.02936
	.99	2.56690	2.88123	3.05955	3.18358	3.27833	3.35481	3.41880	3.47375	3.52185	3.60304	3.66994	3.72678	3.77619	3.81989
18.	.75	.68836	1.08771	1.29732	1.43710	1.54095	1.62306	1.69069	1.74800	1.79761	1.88022	1.94723	2.00342	2.05169	2.09393
	.90	1.33039	1.68429	1.87634	2.00681	2.10497	2.18331	2.24831	2.30374	2.35196	2.43277	2.49877	2.55444	2.60249	2.64472
	.95	1.73407	2.06804	2.25256	2.37918	2.47510	2.55204	2.61614	2.67096	2.71880	2.79919	2.86507	2.92077	2.96893	3.01131
	.99	2.55236	2.86230	3.03799	3.16016	3.25349	3.32882	3.39185	3.44597	3.49334	3.57325	3.63905	3.69492	3.74342	3.78626
19.	.75	.68762	1.08622	1.29532	1.43471	1.53824	1.62010	1.68750	1.74462	1.79406	1.87637	1.94313	1.99911	2.04721	2.08928
	.90	1.32773	1.68004	1.87105	2.00073	2.09826	2.17608	2.24064	2.29566	2.34354	2.42374	2.48923	2.54445	2.59211	2.63398
	.95	1.72913	2.06083	2.24387	2.36939	2.46443	2.54065	2.60412	2.65841	2.70576	2.78533	2.85053	2.90565	2.95332	2.99526
	.99	2.53947	2.84552	3.01884	3.13934	3.23139	3.30568	3.36784	3.42121	3.46791	3.54670	3.61155	3.66659	3.71434	3.75649
20.	.75	.68695	1.08488	1.29352	1.43256	1.53582	1.61744	1.68464	1.74158	1.79087	1.87292	1.93946	1.99525	2.04318	2.08511
	.90	1.32534	1.67624	1.86630	1.99529	2.09226	2.16961	2.23376	2.28844	2.33601	2.41566	2.48070	2.53552	2.58284	2.62439
	.95	1.72472	2.05438	2.23610	2.36064	2.45490	2.53047	2.59339	2.64719	2.69411	2.77295	2.83754	2.89214	2.93935	2.98089
	.99	2.52797	2.83054	3.00174	3.12072	3.21160	3.28495	3.34632	3.39901	3.44512	3.52290	3.58690	3.64120	3.68831	3.72987
24.	.75	.68485	1.08067	1.28786	1.42581	1.52819	1.60908	1.67565	1.73204	1.78084	1.86205	1.92789	1.98308	2.03048	2.07195
	.90	1.31784	1.66429	1.85143	1.97822	2.07344	2.14933	2.21223	2.26581	2.31240	2.39038	2.45400	2.50761	2.55385	2.59444
	.95	1.71088	2.03419	2.21180	2.33329	2.42513	2.49868	2.55987	2.61216	2.65775	2.73429	2.79697	2.84993	2.89571	2.93598
	.99	2.49216	2.78391	2.94844	3.06262	3.14977	3.22008	3.27890	3.32939	3.37358	3.44811	3.50943	3.56144	3.60655	3.64633

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

ρ = .2 (cont.)

ν	P*	K = 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
30.	.75	.68276	1.09550	1.31147	1.45535	1.56220	1.64667	1.71624	1.77521	1.82626	1.91128	1.98026	2.03812	2.08785	2.13137
	.90	1.31042	1.66419	1.85515	1.98459	2.08186	2.15945	2.22380	2.27866	2.32639	2.40634	2.47164	2.52671	2.57424	2.61600
	.95	1.69726	2.02282	2.20150	2.32376	2.41622	2.49033	2.55203	2.60479	2.65081	2.72816	2.79155	2.84516	2.89155	2.93238
	.99	2.45726	2.74293	2.90379	3.01544	3.10072	3.16958	3.22724	3.27679	3.32018	3.39347	3.45388	3.50519	3.54976	3.58911
36.	.75	.68137	1.09262	1.30754	1.45061	1.55679	1.64071	1.70980	1.76834	1.81901	1.90337	1.97180	2.02919	2.07850	2.12165
	.90	1.30551	1.65623	1.84512	1.97299	2.06900	2.14552	2.20896	2.26301	2.31002	2.38872	2.45298	2.50714	2.55387	2.59491
	.95	1.68830	2.00957	2.18542	2.30555	2.39631	2.46900	2.52947	2.58115	2.62622	2.70191	2.76392	2.81634	2.86167	2.90156
	.99	2.43449	2.71310	2.86949	2.97785	3.06051	3.12720	3.18300	3.23093	3.27288	3.34371	3.40205	3.45159	3.49460	3.53257
48.	.75	.67964	1.08904	1.30266	1.44472	1.55008	1.63331	1.70179	1.75980	1.81000	1.89355	1.96129	2.01808	2.06687	2.10955
	.90	1.29944	1.64638	1.83272	1.95865	2.05309	2.12830	2.19060	2.24365	2.28977	2.36694	2.42990	2.48294	2.52868	2.56884
	.95	1.67722	1.99322	2.16560	2.28313	2.37179	2.44273	2.50169	2.55205	2.59593	2.66959	2.72988	2.78081	2.82484	2.86357
	.99	2.40658	2.67660	2.82755	2.93188	3.01134	3.07536	3.12888	3.17481	3.21499	3.28277	3.33855	3.38589	3.42696	3.46320
60.	.75	.67860	1.08690	1.29975	1.44121	1.54608	1.62889	1.69702	1.75471	1.80463	1.88768	1.95501	2.01145	2.05992	2.10232
	.90	1.29582	1.64053	1.82535	1.95013	2.04364	2.11807	2.17969	2.23215	2.27774	2.35399	2.41618	2.46855	2.51370	2.55334
	.95	1.67065	1.98352	2.15386	2.26984	2.35727	2.42717	2.48523	2.53481	2.57799	2.65044	2.70972	2.75977	2.80302	2.84106
	.99	2.39012	2.65511	2.80287	2.90485	2.98243	3.04489	3.09707	3.14182	3.18095	3.24693	3.30119	3.34722	3.38713	3.42235
120.	.75	.67654	1.08265	1.29397	1.43423	1.53813	1.62012	1.68753	1.74459	1.79395	1.87602	1.94253	1.99825	2.04609	2.08793
	.90	1.28865	1.62893	1.81077	1.93327	2.02494	2.09782	2.15811	2.20939	2.25393	2.32836	2.38901	2.44005	2.48403	2.52261
	.95	1.65765	1.96439	2.13070	2.24365	2.32864	2.39649	2.45280	2.50082	2.54263	2.61269	2.66995	2.71826	2.75997	2.79663
	.99	2.35782	2.61305	2.75462	2.85201	2.92594	2.98535	3.03491	3.07736	3.11446	3.17691	3.22820	3.27166	3.30931	3.34250
∞	.75	.67449	1.07844	1.28823	1.42732	1.53025	1.61142	1.67812	1.73456	1.78335	1.86446	1.93013	1.98514	2.03235	2.07362
	.90	1.28155	1.61749	1.79638	1.91665	2.00651	2.07786	2.13683	2.18694	2.23044	2.30307	2.36218	2.41190	2.45470	2.49224
	.95	1.64485	1.94560	2.10797	2.21796	2.30056	2.36640	2.42098	2.46749	2.50793	2.57564	2.63092	2.67750	2.71770	2.75299
	.99	2.32635	2.57216	2.70778	2.80077	2.87118	2.92766	2.97470	3.01495	3.05007	3.10912	3.15755	3.19852	3.23398	3.26521

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .2$

ν	P*	K 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
15.	.75	.69120	1.11309	1.33553	1.48440	1.59531	1.68320	1.75571	1.81726	1.87061	1.95957	2.03185	2.09255	2.14475	2.19048
	.90	1.34061	1.71349	1.91737	2.05664	2.16186	2.24615	2.31629	2.37626	2.42857	2.51649	2.58858	2.64958	2.70239	2.74890
	.95	1.75305	2.10582	2.30246	2.43820	2.54148	2.62458	2.69396	2.75339	2.80528	2.89253	2.96398	3.02428	3.07628	3.12189
	.99	2.60240	2.93348	3.12243	3.25444	3.35577	3.43798	3.50719	3.56702	3.61976	3.70981	3.78527	3.85054	3.90835	3.96044
16.	.75	.69013	1.11086	1.33248	1.48071	1.59110	1.67856	1.75070	1.81192	1.86498	1.95345	2.02532	2.08567	2.13757	2.18302
	.90	1.33676	1.70718	1.90939	2.04738	2.15158	2.23499	2.30437	2.36366	2.41536	2.50220	2.57337	2.63355	2.68563	2.73148
	.95	1.74589	2.09511	2.28940	2.42338	2.52525	2.60719	2.67559	2.73419	2.78537	2.87146	2.94203	3.00166	3.05316	3.09838
	.99	2.58344	2.90864	3.09407	3.22355	3.32286	3.40335	3.47099	3.52935	3.58068	3.66800	3.74074	3.80327	3.85828	3.90752
17.	.75	.68920	1.10890	1.32980	1.47747	1.58741	1.67449	1.74630	1.80724	1.86004	1.94807	2.01958	2.07962	2.13124	2.17645
	.90	1.33338	1.70165	1.90240	2.03928	2.14257	2.22522	2.29394	2.35265	2.40382	2.48974	2.56011	2.61960	2.67105	2.71634
	.95	1.73961	2.08574	2.27798	2.41042	2.51106	2.59200	2.65953	2.71739	2.76793	2.85295	2.92269	2.98165	3.03262	3.07743
	.99	2.56690	2.88694	3.06924	3.19648	3.29403	3.37302	3.43935	3.49651	3.54672	3.63193	3.70266	3.76323	3.81628	3.86356
18.	.75	.68836	1.10717	1.32742	1.47460	1.58414	1.67088	1.74240	1.80308	1.85566	1.94331	2.01449	2.07425	2.12564	2.17063
	.90	1.33039	1.69676	1.89623	2.03213	2.13463	2.21661	2.28475	2.34294	2.39365	2.47877	2.54846	2.60734	2.65825	2.70305
	.95	1.73407	2.07747	2.26792	2.39901	2.49857	2.57860	2.64537	2.70257	2.75253	2.83658	2.90554	2.96387	3.01432	3.05870
	.99	2.55236	2.86784	3.04735	3.17259	3.26856	3.34625	3.41144	3.46758	3.51685	3.60035	3.66951	3.72858	3.78017	3.82602
19.	.75	.68762	1.10562	1.32530	1.47204	1.58122	1.66766	1.73893	1.79938	1.85176	1.93906	2.00995	2.06946	2.12063	2.16543
	.90	1.32773	1.69241	1.89073	2.02576	2.12756	2.20895	2.27658	2.33432	2.38462	2.46904	2.53812	2.59647	2.64691	2.69128
	.95	1.72913	2.07013	2.25898	2.38888	2.48747	2.56671	2.63280	2.68940	2.73884	2.82202	2.89026	2.94800	2.99796	3.04192
	.99	2.53947	2.85090	3.02791	3.15136	3.24592	3.32244	3.38663	3.44188	3.49035	3.57240	3.64027	3.69814	3.74860	3.79335
20.	.75	.68695	1.10423	1.32340	1.46974	1.57860	1.66477	1.73581	1.79606	1.84825	1.93524	2.00587	2.06516	2.11613	2.16076
	.90	1.32534	1.68851	1.88581	2.02006	2.12123	2.20209	2.26926	2.32660	2.37655	2.46033	2.52888	2.58676	2.63679	2.68079
	.95	1.72472	2.06356	2.25099	2.37982	2.47756	2.55608	2.62156	2.67763	2.72660	2.80898	2.87657	2.93377	2.98326	3.02683
	.99	2.52797	2.83578	3.01055	3.13238	3.22567	3.30114	3.36444	3.41890	3.46666	3.54747	3.61423	3.67111	3.72063	3.76451
24.	.75	.68485	1.09984	1.31741	1.46251	1.57036	1.65568	1.72598	1.78559	1.83721	1.92321	1.99302	2.05160	2.10195	2.14602
	.90	1.31784	1.67627	1.87037	2.00219	2.10140	2.18061	2.24635	2.30244	2.35126	2.43311	2.50001	2.55646	2.60521	2.64807
	.95	1.71088	2.04301	2.22601	2.35151	2.44658	2.52287	2.58644	2.64084	2.68832	2.76818	2.83367	2.88909	2.93705	2.97927
	.99	2.49216	2.78872	2.95646	3.07316	3.16244	3.23459	3.29507	3.34706	3.39263	3.46963	3.53316	3.58716	3.63410	3.67558

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV (cont.)

$\rho = .1$ (cont.)

ν	P* \ K	1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
30.	.75	.68276	1.11194	1.33655	1.48632	1.59766	1.68578	1.75842	1.82004	1.87344	1.96245	2.03476	2.09548	2.14771	2.19346
	.90	1.31042	1.67354	1.86971	2.00285	2.10305	2.18308	2.24953	2.30624	2.35562	2.43845	2.50621	2.56343	2.61287	2.65636
	.95	1.69726	2.02922	2.21162	2.33660	2.43126	2.50722	2.57053	2.62473	2.67205	2.75167	2.81702	2.87235	2.92026	2.96247
	.99	2.45726	2.74586	2.90858	3.02167	3.10817	3.17809	3.23670	3.28712	3.33133	3.40609	3.46783	3.52036	3.56606	3.60649
36.	.75	.68137	1.10895	1.33240	1.48125	1.59183	1.67930	1.75137	1.81249	1.86544	1.95366	2.02531	2.08546	2.13718	2.18247
	.90	1.30551	1.66541	1.85933	1.99076	2.08955	2.16839	2.23380	2.28959	2.33816	2.41957	2.48612	2.54228	2.59080	2.63345
	.95	1.68830	2.01576	2.19515	2.31784	2.41064	2.48505	2.54701	2.60002	2.64628	2.72407	2.78788	2.84188	2.88863	2.92981
	.99	2.43449	2.71582	2.87389	2.98353	3.06726	3.13486	3.19149	3.24016	3.28281	3.35488	3.41432	3.46486	3.50879	3.54762
48.	.75	.67964	1.10524	1.32725	1.47495	1.58459	1.67125	1.74262	1.80311	1.85549	1.94273	2.01355	2.07297	2.12405	2.16877
	.90	1.29944	1.65534	1.84651	1.97581	2.07286	2.15022	2.21436	2.26902	2.31657	2.39621	2.46127	2.51613	2.56349	2.60511
	.95	1.67722	1.99916	2.17485	2.29473	2.38526	2.45775	2.51805	2.56960	2.61455	2.69007	2.75195	2.80429	2.84957	2.88944
	.99	2.40658	2.67906	2.83148	2.93691	3.01726	3.08205	3.13625	3.18279	3.22353	3.29230	3.34896	3.39708	3.43887	3.47578
60.	.75	.67860	1.10303	1.32417	1.47120	1.58027	1.66645	1.73739	1.79750	1.84955	1.93620	2.00651	2.06550	2.11619	2.16057
	.90	1.29582	1.64935	1.83888	1.96692	2.06294	2.13943	2.20280	2.25679	2.30373	2.38233	2.44649	2.50058	2.54725	2.58825
	.95	1.67065	1.98931	2.16283	2.28105	2.37023	2.44158	2.50090	2.55158	2.59575	2.66992	2.73067	2.78201	2.82642	2.86549
	.99	2.39012	2.65743	2.80654	2.90951	2.98789	3.05102	3.10380	3.14908	3.18871	3.25555	3.31057	3.35727	3.39780	3.43358
120.	.75	.67654	1.09863	1.31806	1.46374	1.57168	1.65690	1.72700	1.78636	1.83772	1.92320	1.99250	2.05061	2.10052	2.14419
	.90	1.28865	1.63750	1.82380	1.94934	2.04332	2.11807	2.17994	2.23259	2.27833	2.35483	2.41721	2.46975	2.51505	2.55481
	.95	1.65765	1.96990	2.13912	2.25408	2.34061	2.40973	2.46711	2.51608	2.55872	2.63022	2.68870	2.73808	2.78074	2.81825
	.99	2.35782	2.61509	2.75778	2.85597	2.93052	2.99045	3.04046	3.08332	3.12077	3.18385	3.23569	3.27962	3.31771	3.35129
∞	.75	.67449	1.09426	1.31201	1.45634	1.56317	1.64742	1.71668	1.77529	1.82598	1.91027	1.97855	2.03577	2.08489	2.12785
	.90	1.28155	1.62581	1.80893	1.93201	2.02397	2.09702	2.15739	2.20871	2.25326	2.32769	2.38830	2.43928	2.48320	2.52173
	.95	1.64485	1.95082	2.11585	2.22762	2.31157	2.37849	2.43398	2.48126	2.52240	2.59128	2.64753	2.69496	2.73589	2.77185
	.99	2.32635	2.57395	2.71048	2.80409	2.87497	2.93183	2.97920	3.01972	3.05509	3.11457	3.16335	3.20464	3.24037	3.27185

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

TABLE IV

Upper 100α Percentage Point of the Studentized Maximum of Equicorrelated Standard Normal Variables

$\rho = .1$

ν	P*	K 1.	2.	3.	4.	5.	6.	7.	8.	9.	11.	13.	15.	17.	19.
15.	.75	.69120	1.13019	1.36196	1.51737	1.63337	1.72544	1.80152	1.86617	1.92229	2.01602	2.09234	2.15657	2.21192	2.26052
	.90	1.34061	1.72395	1.93410	2.07805	2.18708	2.27461	2.34758	2.41007	2.46464	2.55647	2.63183	2.69558	2.75072	2.79919
	.95	1.75305	2.11354	2.31507	2.45455	2.56085	2.64649	2.71801	2.77928	2.83277	2.92260	2.99603	3.05784	3.11102	3.15753
	.99	2.60240	2.93786	3.13005	3.26497	3.36910	3.45409	3.52607	3.58867	3.64421	3.73982	3.82077	3.89138	3.95429	4.01119
16.	.75	.69013	1.12788	1.35873	1.51343	1.62883	1.72041	1.79605	1.86033	1.91610	2.00923	2.08504	2.14881	2.20375	2.25195
	.90	1.33676	1.71749	1.92584	2.06839	2.17627	2.26282	2.33493	2.39666	2.45055	2.54120	2.61558	2.67851	2.73296	2.78087
	.95	1.74589	2.10265	2.30169	2.43928	2.54409	2.62852	2.69904	2.75948	2.81228	2.90105	2.97373	3.03505	3.08790	3.13423
	.99	2.58344	2.91282	3.10127	3.23338	3.33516	3.41804	3.48804	3.54874	3.60240	3.69434	3.77164	3.83864	3.89798	3.95140
17.	.75	.68920	1.12584	1.35590	1.50996	1.62485	1.71599	1.79125	1.85519	1.91066	2.00327	2.07863	2.14200	2.19659	2.24446
	.90	1.33338	1.71184	1.91860	2.05993	2.16681	2.25250	2.32387	2.38494	2.43823	2.52786	2.60138	2.66358	2.71741	2.76477
	.95	1.73961	2.09313	2.28999	2.42594	2.52943	2.61278	2.68241	2.74209	2.79424	2.88199	2.95392	3.01468	3.06714	3.11318
	.99	2.56690	2.89095	3.07609	3.20575	3.30552	3.38663	3.45502	3.51419	3.56640	3.65553	3.73011	3.79445	3.85119	3.90205
18.	.75	.68836	1.12404	1.35339	1.50690	1.62133	1.71207	1.78700	1.85064	1.90584	1.99798	2.07295	2.13598	2.19025	2.23784
	.90	1.33039	1.70684	1.91221	2.05246	2.15846	2.24340	2.31412	2.37461	2.42738	2.51611	2.58887	2.65042	2.70368	2.75055
	.95	1.73407	2.08473	2.27967	2.41417	2.51651	2.59890	2.66772	2.72671	2.77827	2.86505	2.93625	2.99645	3.04847	3.09418
	.99	2.55236	2.87170	3.05390	3.18140	3.27941	3.35900	3.42603	3.48395	3.53496	3.62186	3.69433	3.75663	3.81138	3.86031
19.	.75	.68762	1.12244	1.35115	1.50416	1.61818	1.70858	1.78320	1.84657	1.90154	1.99327	2.06789	2.13061	2.18461	2.23195
	.90	1.32773	1.70239	1.90652	2.04581	2.15103	2.23531	2.30546	2.36543	2.41775	2.50567	2.57776	2.63872	2.69147	2.73791
	.95	1.72913	2.07727	2.27051	2.40373	2.50503	2.58656	2.65465	2.71302	2.76404	2.84993	2.92042	2.98007	3.03165	3.07701
	.99	2.53947	2.85463	3.03422	3.15973	3.25624	3.33451	3.40036	3.45720	3.50722	3.59227	3.66303	3.72372	3.77691	3.82434
20.	.75	.68695	1.12099	1.34914	1.50171	1.61536	1.70545	1.77980	1.84293	1.89768	1.98904	2.06334	2.12579	2.17954	2.22666
	.90	1.32534	1.69840	1.90143	2.03987	2.14439	2.22808	2.29770	2.35722	2.40913	2.49634	2.56782	2.62827	2.68056	2.72659
	.95	1.72472	2.07060	2.26232	2.39439	2.49477	2.57553	2.64296	2.70077	2.75128	2.83635	2.90619	2.96530	3.01645	3.06146
	.99	2.52797	2.83940	3.01663	3.14047	3.23554	3.31263	3.37745	3.43337	3.48253	3.56602	3.63536	3.69472	3.74666	3.79287
24.	.75	.68485	1.11645	1.34282	1.49398	1.60647	1.69557	1.76907	1.83144	1.88551	1.97570	2.04900	2.11059	2.16358	2.21001
	.90	1.31784	1.68589	1.88545	2.02122	2.12356	2.20540	2.27342	2.33152	2.38215	2.46715	2.53675	2.59556	2.64642	2.69117
	.95	1.71088	2.04972	2.23672	2.36520	2.46269	2.54103	2.60640	2.66239	2.71131	2.79368	2.86132	2.91861	2.96822	3.01191
	.99	2.49216	2.79199	2.96187	3.08028	3.17102	3.24448	3.30614	3.35925	3.40586	3.48482	3.55015	3.60586	3.65442	3.69746

The entry in each case is the value of y^* satisfying $G_{k,\nu}(y^*,\rho) = P^* = 1-\alpha$, where $G_{k,\nu}(y^*,\rho)$ is given by (2.3).

6. APPLICATIONS

In this section we discuss briefly some specific applications illustrating the use of the tables. These applications relate to selection procedures, multiple comparisons, prediction intervals and tests of hypotheses.

6.1 Selection procedures

Let $\pi_1,\ldots,\pi_k$ be k independent normal populations with unknown means $\mu_1,\ldots,\mu_k$, respectively, and common unknown variance σ^2. Gupta [5], [7] proposed a procedure R_1 for selecting a nonempty subset from the k populations so that the population associated with the largest μ_i is included in the selected subset (this event is called a correct selection (CS)) with a probability at least equal to P* ($1/k < P^* < 1$). Let $Y_i = \bar{X}_i$, $i = 1,\ldots,k$, be the means of samples of size n from these populations. The rule R_1 is: Select π_i if and only if $Y_i \geq \max_{1\leq j\leq k} Y_j - \frac{ds}{\sqrt{n}}$, where s^2 is the usual pooled estimator of σ^2 based on all the k samples such that $\nu s^2/\sigma^2$ has a χ^2 distribution with $\nu = k(n-1)$ degrees of freedom. The infimum of PCS, the probability of a CS, occurs when $\mu_1 = \ldots = \mu_k$ for all values of $\sigma^2 > 0$: Thus the value of d satisfying the probability requirement is given by $G_{k-1,\nu}(d/\sqrt{2};\ 1/2) = P^*$. Therefore, $d = \sqrt{2}\ y^*(1-P^*,k-1,k(n-1),0.5)$.

Suppose the above k populations represent experimental categories and they are compared with a control population π_0 which is $N(\mu_0,\sigma^2)$ where μ_0 is unknown. Let $Y_0 = \bar{X}_0$ be the mean of a sample of size m from π_0. The pooled estimator s^2 is now based on $\nu = k(n-1)+(m-1)$ degrees of freedom. Gupta and Sobel [10] proposed a rule R_2 for selecting a subset of the k experimental populations such that all populations better than the control (i.e. those for which $\mu_i \geq \mu_0$) are included in the selected subset with a probability at least equal to P*. We propose a modified procedure R_2':

Select π_i if and only if $\bar{X}_i \geq \bar{X}_0 - Ds\sqrt{\frac{1}{m} + \frac{1}{n}}$, where $D > 0$ is to be determined to satisfy the probability requirement. It is easy to show that the infimum of PCS is equal to $G_{k,\nu}(D; \rho)$ where $\rho = n/(n+m)$. This gives $D = y^*(1-P^*,k,\nu,\rho)$.

6.2 Multiple comparisons between k treatment means and a control mean

The problem of the simultaneous one-sided comparisons of the means of k treatments with that of a control group has been discussed by Dunnett [3]. Let $\bar{X}_0,\bar{X}_1,\ldots,\bar{X}_k$ be independent sample means based on m,n,...,n independent observations from normal populations with unknown means $\mu_0,\mu_1,\ldots,\mu_k$, respectively, and a common unknown variance σ^2. Let s^2 be the usual pooled unbiased estimator of σ^2 based on $\nu = k(n-1)+(m-1)$ degrees of freedom such that $\nu s^2/\sigma^2$ has a χ^2-distribution with ν degrees of freedom. Then a set of k exact one-sided (lower) 100γ% simultaneous confidence intervals for the differences $\mu_i-\mu_0$, $i = 1,\ldots,k$, is given by

$$(\bar{X}_i-\bar{X}_0) - y^*(1-\gamma,k,\nu,\rho)\ s\sqrt{\frac{1}{m} + \frac{1}{n}}, \quad i = 1,\ldots,k$$

where $\rho = n/(n+m)$.

6.3 Prediction intervals to contain all of k future means

Let $Y_1,\ldots,Y_n$ be the n observations of a current sample from a normal population with mean μ and variance σ^2, both unknown. Let $\bar{Y}$ be the sample mean and s^2 be the usual unbiased estimator of σ^2 on $\nu = n-1$ degrees of freedom. Let $\bar{X}_1,\ldots,\bar{X}_k$ be the (unknown) sample means of k additional future independent samples each of size m. The $\bar{X}_i$ are independent of $\bar{Y}$ and s^2. An exact lower simultaneous 100γ% prediction interval for the sample means $\bar{X}_1,\ldots,\bar{X}_k$ is given by $\bar{Y} - y^*(1-\gamma,k,\nu,\rho)s\sqrt{\frac{1}{m} + \frac{1}{n}}$, where $\rho = m/(m+n)$.

6.4 Test procedures for multiple comparisons

Krishnaiah [12] proposed test procedures for the multiple comparisons of means (mean vectors) against one-sided alternatives under a general analysis of variance (multivariate analysis of variance) model. Let us consider k normal populations with unknown means $\mu_1,\ldots,\mu_k$ and a common unknown variance σ^2. Consider the problem of testing

H_i: $\underset{\sim}{c}_i'\underset{\sim}{\mu} = 0$ simultaneously against the respective alternatives

K_i: $\underset{\sim}{c}_i'\underset{\sim}{\mu} > 0$, $i = 1,\ldots,q$ $(q < k)$, where $\underset{\sim}{\mu}' = (\mu_1,\ldots,\mu_k)$,

and the $\underset{\sim}{c}_i$ are kx1 vectors of known elements such that $\underset{\sim}{c}_i'\underset{\sim}{c}_i = 1$, $\underset{\sim}{c}_i'\,\underset{-}{1} = 0$ for $i = 1,\ldots,q$, and $\underset{\sim}{c}_i'\underset{\sim}{c}_j = \rho$ (> 0) for $i \neq j = 1,\ldots,q$. Here $\underset{-}{1}$ denotes a column vector whose elements are unity. It is required that

$$\text{pr}\{\text{any } H_i \text{ is rejected} \mid \bigcap_{i=1}^{q} H_i\} = \alpha.$$

Let $\bar{Y}_i$, $i = 1,\ldots,k$, the sample means based on n independent observations from each population and let s^2 denote the usual pooled unbiased estimator of σ^2 on $\nu = k(n-1)$ degrees of freedom. Then, as a special case of the procedure proposed by Krishnaiah [12], we use the procedure:

Reject H_i (against K_i) if $t_i = \dfrac{\sqrt{n}\ \underset{\sim}{c}_i'\underset{\sim}{Z}}{s} > t_\alpha$

where $\underset{\sim}{Z} = (\bar{Y}_1,\ldots,\bar{Y}_k)$ and t_α satisfying the error restriction is given by $t_\alpha = y^*(1-\alpha,k,\nu,\rho)$.

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Let $X_1,\ldots,X_k$ have a joint k-variate normal distribution with zero means, common unknown variance σ^2 and known correlation matrix (ρ_{ij}), where $\rho_{ij} = \rho$ for all $i \neq j$. Let s^2 be distributed independently of the X_i such that $\nu s^2/\sigma^2$ has a chi-squared distribution with ν degrees of freedom. New tables with wider coverage and more accuracy than the previously published ones are given for the percentage points of

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$Y = \max_{1 \leq i \leq k} \frac{X_i}{s}$. Some basic theoretical results are given in Section 2. The next section describes Hartley's results for approximating the distribution function of Y. Besides a brief review of existing tables (Section 4), the paper discusses the construction of new tables based on Hartley's results (Section 5) and some specific applications (Section 6).