Prediction Intervals in Balanced One-Factor Random Model

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A B S T R A C T

An exact and three approximate prediction intervals for the over-all mean of a future sample are obtained under a one-factor random model. The validity of these approximate intervals are assessed by a simulation study. A sensitivity comparison is also made.

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PREDICTION INTERVALS IN BALANCED ONE-FACTOR RANDOM MODEL

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1. Introduction.

Prediction intervals for the overall mean of future observations have been considered in the literature under the usual regression (fixed effects) model; see for example, Graybill [2, pp. 267-270] and Hahn [4]. Such problems for random effects models are also of interest in practice; however, this seems to have not received enough attention. The purpose of the present paper is to consider prediction intervals in a balanced one-factor random model given by

$$(1.1) \quad Y_{ij} = \mu + A_i + \varepsilon_{ij}, \quad i = 1,\ldots,I; \; j = 1,\ldots,J,$$

involving I levels of a factor and J observations per level. Here the ε_{ij} are i.i.d. $N(0,\sigma_\varepsilon^2)$, and the A_i are independent of the ε_{ij} and are i.i.d. $N(0,\sigma_A^2)$. The parameters μ, σ_A^2, and σ_ε^2 are all unknown. Let Y_{ij}^* denote the observations in a future experiment according to the model (1.1) where the factor is now taken at I^* levels with J^* observations per level. We are interested in obtaining two-sided prediction intervals for $\bar{\bar{Y}}^*$, the overall mean of the future observations based on the present data. In other words, we are interested in defining an interval (L(Y),U(Y)) such that

(1.2) $$\Pr\{L(Y) \le \bar{\bar{Y}}^* \le U(Y)\} = 1-\alpha$$

where Y denotes the set observations Y_{ij}, and the prediction level $1-\alpha$ is specified in advance. The limits L(Y) and U(Y) will, of course, depend on Y through $\bar{\bar{Y}} = \sum_i \sum_j Y_{ij}/IJ$, $S_1^2 = J \sum_i (\bar{Y}_i - \bar{\bar{Y}})^2/I-1$, and $S_2^2 = \sum_i \sum_j (Y_{ij} - \bar{Y}_i)^2/I(J-1)$, since $(\bar{\bar{Y}}, S_1^2, S_2^2)$ is minimal sufficient for $(\mu, \sigma_A^2, \sigma_\varepsilon^2)$.

In dealing with prediction interval for $\bar{\bar{Y}}^*$, several cases arise. When the ratio $\sigma_A^2/\sigma_\varepsilon^2$, or equivalently, $R = (\sigma_\varepsilon^2 + J\sigma_A^2)/\sigma_\varepsilon^2$, is assumed to be known, an exact prediction interval is obtained (Section 3.1). When R is unknown, an exact interval is obtained only when $J^* \le J$ (Section 3.2). This interval based on certain linear combinations of the Y_{ij} is not unique; however, in the particular case of $J^* = J$, this method gives a unique interval based on the minimal sufficient statistics. For the case of unknown R, approximate intervals (in the sense that the prediction level is approximately $1-\alpha$) obtained by different methods, namely, (i) Plug-in Method (Section 4.1), (ii) Modified Large Sample Method (Section 4.2), and (iii) Satterthwaite Approximation Method (Section 4.3) are discussed. The relative performances of these different procedures are evaluated by a simulation study (Section 5), which finds the modified large sample method most satisfactory in terms of validity. In view of these, the approximate interval using this method is compared with the exact interval which is available when $J^* \le J$. It is shown (Section 6) that the approximate method is superior in terms of the expected squared length unless $J^* = J$ in which case both yield the same (exact) interval.

2. Notations and Preliminary Results

In this section, we introduce a set of notations that will be often used throughout the paper. We will also state a few well-known results that will be repeatedly used. Let

$$(2.1)\quad \begin{cases} n_1 = I - 1, & n_2 = I(J - 1), \\ M_1 = \frac{1}{J}\left[\frac{1}{I^*} + \frac{1}{I}\right], & M_2 = \frac{1}{I^*}\left[\frac{1}{J^*} - \frac{1}{J}\right], \\ \tau_1^2 = \sigma_\varepsilon^2 + J\sigma_A^2, & \tau_2^2 = \sigma_\varepsilon^2, \\ \rho = \sigma_A^2/(\sigma_\varepsilon^2 + \sigma_A^2), & R = (\sigma_\varepsilon^2 + J\sigma_A^2)/\sigma_\varepsilon^2 = \tau_1^2/\tau_2^2, \\ \tau^2 = \mathrm{Var}(\bar{\bar{Y}}^* - \bar{\bar{Y}}) = \tau_2^2[M_2 + RM_1] = M_1\tau_1^2 + M_2\tau_2^2, & \end{cases}$$

(2.1) (Cont.)

$$\delta_1^2(\rho) = \tau_1^2/\tau^2 = \frac{1 + (J-1)\rho}{M_1[1 + (J-1)\rho] + M_2(1-\rho)},$$

$$\delta_2^2(\rho) = \tau_2^2/\tau^2 = \frac{1 - \rho}{M_1[1 + (J-1)\rho] + M_2(1-\rho)},$$

$$U_1 = \frac{(I-1)S_1^2}{\sigma_\varepsilon^2 + J\sigma_A^2} = \frac{n_1 S_1^2}{\tau_1^2}, \quad U_2 = \frac{I(J-1)S_2^2}{\sigma_\varepsilon^2} = \frac{n_2 S_2^2}{\tau_2^2},$$

$$S_p^2 = \frac{n_2 S_2^2 + (n_1 S_1^2/R)}{n_1 + n_2},$$

$$u^+ = u \quad \text{if} \quad u > 0, \quad \text{and} \quad = 0, \quad \text{otherwise.}$$

Let $N(\mu,\sigma^2)$, χ_ν^2 and t_ν denote the normal distribution with mean μ and variance σ^2, chi-square distribution with ν degrees of freedom (d.f.), and Student's t-distribution with ν d.f., respectively. Further, z_γ and $t_{\gamma,\nu}$ denote the upper γ quantiles of N(0,1) and t_ν distributions, respectively. As usual, E(·) denotes the expectation of a random variable.

We conclude this section by stating the following results which are easily verified.

1. $\bar{\bar{Y}} \sim N(\mu, \tau_1^2/IJ)$.
2. $U_1 \sim \chi_{n_1}^2$ and $U_2 \sim \chi_{n_2}^2$.
3. $\bar{\bar{Y}}$, U_1 and U_2 are independent.
4. $\bar{\bar{Y}}^* - \bar{\bar{Y}} \sim N(0, \tau^2)$.
5. $(n_1 + n_2)S_p^2/\tau_2^2 \sim \chi_{n_1+n_2}^2$.

3. Exact Prediction Intervals

We consider two cases: A. R known and B. R unknown. In Case B, we assume that $J^* \le J$.

3.1 Case A: R Known. Using the fact that

$$\frac{\bar{\bar{Y}}^* - \bar{\bar{Y}}}{S_p\sqrt{M_2 + RM_1}} \sim t_{n_1+n_2},$$

we get an exact $(1-\alpha)$ level prediction interval for $\bar{\bar{Y}}^*$ given by

$$(3.1) \qquad I_{ER}: \bar{\bar{Y}} \pm t_{\alpha/2, n_1+n_2} S_p\sqrt{M_2 + RM_1}.$$

3.2 Case B: R Unknown and $J^* \le J$. Let us consider $D_i = \sum_{j=1}^{J} \ell_{ij} Y_{ij}$, $i = 1, \ldots, I$, where the coefficients ℓ_{ij} are chosen such that, for all $i = 1, \ldots, I$,

$$\sum_{j=1}^{J} \ell_{ij} = 1 \quad \text{and} \quad \text{Var}\,(D_i) = (\sigma_\varepsilon^2 + J^*\sigma_A^2)/J^* = \sigma_D^2, \text{ say.}$$

Since $\text{Var}\,(D_i) = \sigma_A^2\left[\sum_j \ell_{ij}\right]^2 + \sigma_\varepsilon^2 \sum_j \ell_{ij}^2$, one possible choice is $\ell_{ij} = \frac{1}{J^*}$, $j = 1, \ldots, J^*$, and $\ell_{ij} = 0$ otherwise. Now, let $\bar{D} = \sum_{i=1}^{I} D_i/I$. The D_i are i.i.d. $N(\mu, \sigma_D^2)$. Noting that

$\sum_{i=1}^{I} (D_i - \bar{D})^2/\sigma_D^2 \sim \chi^2_{n_1}$, it is easy to show that

$$\frac{\bar{\bar{Y}}^* - \bar{D}}{\sqrt{\sum (D_i - \bar{D})^2}} \sqrt{\frac{I(I-1)I^*}{(I + I^*)}} \sim t_{n_1}.$$

Thus we get an exact $(1-\alpha)$ level prediction interval for $\bar{\bar{Y}}^*$ given by

$$(3.2) \qquad I_{ED}: \ \bar{D} \pm t_{\alpha/2, n_1} \sqrt{\frac{I + I^*}{II^*} \ \frac{\sum (D_i - \bar{D})^2}{I - 1}}.$$

The idea of taking suitable linear combination of observations has been used by Burdick and Sielken [1] in the context of confidence intervals in variance components models.

Remark 3.1. It should be noted that the choice of the coefficients ℓ_{ij} in D_i is not unique unless $J^* = J$. When $J^* = J$, a unique interval I_{ED} is obtained based on minimal sufficient statistics. When $J^* < J$, one would expect I_{ED} to be less efficient as it does not depend on minimal sufficient statistics. In other words, we pay a price in obtaining an exact interval. This is brought out in Section 6 where I_{ED} and I_{AM} (to be defined in Section 4.2) are compared in terms of expected squared length.

4. Approximate Prediction Intervals

When R is known, we have an exact prediction interval for $\bar{\bar{Y}}^*$ which is easy to calculate. The need for approximate intervals arise when R is unknown. We consider three different methods, namely, (A) Plug-in Method, (B) Modified Large Sample Method, and (C) Satterthwaite Approximation Method.

4.1 Plug-in Method. Since $R = \tau_1^2/\tau_2^2 > 1$, we define $\hat{R} = \max(1, S_1^2/S_2^2)$ and obtain an approximately $(1-\alpha)$ level prediction interval for $\bar{\bar{Y}}^*$ given by

$$(4.1) \qquad I_{AP}: \quad \bar{\bar{Y}} \pm t_{\alpha/2, n_1+n_2}\, \hat{S}_p \sqrt{M_2 + \hat{R}M_1},$$

where $\hat{S}_p$ is obtained by replacing R with $\hat{R}$ in S_p.

4.2 Modified Large Sample Method. This method is motivated by Graybill and Wang [3] who used such a procedure to obtain good approximate confidence intervals on nonnegative linear combinations of variances. The main idea is to modify a large sample interval so that it might be more exact for small or moderate sample sizes.

In our problem, as $I \to \infty$, we see that

$$\frac{\bar{\bar{Y}}^* - \bar{\bar{Y}}}{\sqrt{(M_1S_1^2 + M_2S_2^2)^+}} \sim N(0,1).$$

So a large sample $(I \to \infty)$ prediction interval for $\bar{\bar{Y}}^*$ is given by

$$(4.2) \qquad \bar{\bar{Y}} \pm \sqrt{(z^2_{\alpha/2} M_1S_1^2 + z^2_{\alpha/2} M_2S_2^2)^+},$$

which has an associated coverage probability $1-\alpha$ asymptotically. Now, the modified large sample method considers the interval (4.2) with the first $z^2_{\alpha/2}$ under the radical sign replaced by a constant A; this constant A is to be determined such that the associated coverage probability tends to $1-\alpha$ as $\sigma^2_A \to \infty$. This yields the prediction interval

$$(4.3) \qquad I_{AM}: \ \bar{\bar{Y}} \pm \sqrt{(t^2_{\alpha/2, n_1} M_1S_1^2 + z^2_{\alpha/2} M_2S_2^2)^+}.$$

It should be noted that the interval I_{AM} is exact when $J^* = J$.

4.3 Satterthwaite Approximation Method. We first note that $Q = M_1S_1^2 + M_2S_2^2$ is an unbiased estimator of $\tau^2 = \mathrm{Var}\,(\bar{\bar{Y}}^* - \bar{\bar{Y}})$. However, Q can be negative when $J^* > J$; in this case, we will use a modified estimator. We discuss the two cases separately.

<u>Case (i): $J^* \le J$.</u> We want to find a nonnegative function $k(S_1^2, S_2^2)$ such that

$$\text{(4.4)} \qquad \Pr[\,|\bar{\bar{Y}}^* - \bar{\bar{Y}}| \le k(S_1^2, S_2^2)\sqrt{Q}\,] = 1-\alpha.$$

Now, Q is a linear combination of independent but not necessarily identical chi-square variables. The Satterthwaite [6] approximation gives

$$\text{(4.5)} \qquad NQ/E(Q) \;\approx\; \chi_N^2$$

where

$$\text{(4.6)} \qquad N = 2[E(Q)]^2/\mathrm{Var}\,(Q).$$

However, we do not know N as it depends on γ_1^2 and γ_2^2, unless $J^* = J$. So we use the estimates S_1^2 and S_2^2, respectively, leading to

$$\text{(4.7)} \qquad \hat{N} = [M_1S_1^2 + M_2S_2^2]^2 / \left[\frac{M_1^2S_1^4}{n_1} + \frac{M_2^2S_2^4}{n_2}\right].$$

Using the usual method of constructing a t-variable from a standard normal variate and a chi-square variate, we get an approximate $(1-\alpha)$ level prediction interval for $\bar{\bar{Y}}^*$ given by

$$\text{(4.8)} \qquad I_{AS}\colon\; \bar{\bar{Y}} \pm t_{\alpha/2,\hat{N}}\,\sqrt{Q}.$$

When $J^* = J$, we have $Q = M_1 S_1^2$ and $\hat{N} = n_1$. In this case, $\hat{N}Q/E(Q)$ is exactly distributed as $\chi^2_{n_1}$ and consequently, I_{AS} is exact.

Case (ii): $J^* > J$. Note that Q can be written in the form

(4.9) $$Q = \frac{S_1^2}{IJ} + \frac{S_2^2}{I^*J^*} + \frac{(S_1^2 - S_2^2)}{I^*J} .$$

When $J^* > J$, we use the interval (4.7) with Q^+ in the place of Q, where

(4.10) $$Q^+ = \frac{S_1^2}{IJ} + \frac{S_2^2}{I^*J^*} + \frac{(S_1^2 - S_2^2)^+}{I^*J} .$$

Obviously, we can combine the two cases and write the prediction interval as

(4.11) $$I_{AS}: \; \bar{\bar{Y}} \pm t_{\alpha/2,\hat{N}} \sqrt{Q^+}$$

5. Validity of Approximate Intervals: Simulation Study

The prediction intervals I_{AP}, I_{AM}, and I_{AS} are constructed so that the coverage probability for each is approximately the specified level. The validity of any of these intervals depend on how close the actual coverage probability is to the specified level. This was investigated by a simulation study. In order to facilitate the estimation of the coverage probabilities, we provide below alternate forms for them. These can be easily verified by using the results in Section 2. Let $P(I_{AP})$, $P(I_{AM})$ and $P(I_{AS})$ denote the coverage probabilities of I_{AP}, I_{AM} and I_{AS}, respectively. Let $Z \sim N(0,1)$. Then the coverage probabilities can be written in the form

$$(5.1) \qquad P(I) = \Pr[\,|Z| \le \Psi(U_1, U_2;\ M_1, M_2, \rho, \alpha)]$$

where the Ψ-functions for the intervals (labeled accordingly) are as follows:

$$\Psi_{AP} = \begin{cases} t_{\frac{\alpha}{2}, n_1+n_2}\sqrt{\dfrac{M_1U_1\delta_1^2(\rho)}{n_1} + \dfrac{M_2U_2\delta_2^2(\rho)}{n_2}} & \text{if } \dfrac{n_2U_1\delta_1^2}{n_1U_2\delta_2^2} \ge 1 \\[2ex] t_{\frac{\alpha}{2}, n_1+n_2}\sqrt{\dfrac{(M_1 + M_2)[U_1\delta_1^2(\rho) + U_2\delta_2^2(\rho)]}{(n_1 + n_2)}} & \text{otherwise;} \end{cases}$$

$$\Psi_{AM} = \sqrt{\left[t^2_{\alpha/2, n_1}\frac{M_1U_1\delta_1^2(\rho)}{n_1} + z^2_{\alpha/2}\frac{M_2U_2\delta_2^2(\rho)}{n_2}\right]^+};$$

and

$$
\hat{I}_{AS} = \begin{cases} t_{\alpha/2,\hat{N}} \sqrt{\dfrac{M_1U_1\delta_1^2(\rho)}{n_1} + \dfrac{M_2U_2\delta_2^2(\rho)}{n_2}} & \text{if } J^* \le J, \\ \\ t_{\alpha/2,\hat{N}} \sqrt{\dfrac{U_1\delta_1^2(\rho)}{n_1IJ} + \dfrac{U_2\delta_2^2(\rho)}{n_2I^*J^*} + \dfrac{1}{I^*J}\left[\dfrac{U_1\delta_1^2(\rho)}{n_1} - \dfrac{U_2\delta_2^2(\rho)}{n_2}\right]^+} & \text{otherwise} \end{cases}
$$

where

$$
\hat{N} = \left[\frac{M_1U_1\delta_1^2(\rho)}{n_1} + \frac{M_2U_2\delta_2^2(\rho)}{n_2}\right]^2 \Big/ \left[\frac{M_1^2U_1^2\delta_1^4(\rho)}{n_1^3} + \frac{M_2^2U_2^2\delta_2^4(\rho)}{n_2^3}\right].
$$

In order to estimate the coverage probabilities by simulation, 72 different sets of values of (I, J, I^*, J^*) were considered. These are the sets obtained when $I, J = 3, 5, 7$; and $I^*, J^* = 5, 10, 15$, omitting sets with $J = J^*$ (in which case, the intervals are exact). For each of these 72 sets of values, 1000 random sets of observations (Z, U_1, U_2) were generated by using IMSL subroutines. Based on these, the coverage probabilities of the three intervals were estimated for ρ = 0.1 (0.1) 0.9; and α = .05, .10. In the case of I_{AS}, $\hat{N}$ value was rounded down to an integer value. The range of the estimated probabilities over the chosen values of ρ are given in Table 1 only for α = 0.05, since the pattern of the intervals is similar when α = .10.

Insert Table

The results of the above study indicate beyond doubt that the modified large sample method is the most satisfactory in the sense that the actual coverage probability is quite close to the specified level over the entire range of the tables. Obviously, its competitors do not exhibit this behavior; in fact, sometimes they miss the mark by considerable margin.

6. Sensitivity Comparison of I_{ED} and I_{AM}

From the validity point of view, we saw in Section 5 that I_{AM} was the most satisfactory approximate procedure. It still remains to compare I_{AM} with the exact procedure I_{ED} with regard to sensitivity. We will consider the expected squared length as our criterion. Letting L_{ED} and L_{AM} denote the half-lengths of these intervals, we will show that $E[L_{AM}^2] < E[L_{ED}^2]$ when $J^* < J$. Of course, the intervals are identical when $J^* = J$. Since $J^* < J$,

$$E[L_{AM}^2] = t_{\alpha/2,n_1}^2 M_1\gamma_1^2 + z_{\alpha/2}^2 M_2\gamma_2^2$$

$$= \sigma_\varepsilon^2\left[t_{\alpha/2,n_1}^2\left[\frac{1}{I} + \frac{1}{I^*}\right]\frac{1}{J} + z_{\alpha/2}^2\left[\frac{1}{J^*} - \frac{1}{J}\right]\frac{1}{I^*}\right]$$

$$+ \sigma_A^2 t_{\alpha/2,n_1}^2\left[\frac{1}{I} + \frac{1}{I^*}\right]$$

$$< \sigma_\varepsilon^2 t_{\alpha/2,n_1}^2\left[\frac{1}{IJ} + \frac{1}{I^*J^*}\right] + \sigma_A^2 t_{\alpha/2,n_1}^2\left[\frac{1}{I} + \frac{1}{I^*}\right],$$

since $t_{\alpha/2,n_1}^2 > z_{\alpha/2}^2$. Now, by replacing J on the right-hand side by J^*, we get

$$E[L_{AM}^2] < \sigma_\varepsilon^2 t_{\alpha/2,n_1}^2\left[\frac{1}{I} + \frac{1}{I^*}\right]\frac{1}{J^*} + \sigma_A^2 t_{\alpha/2,n_1}^2\left[\frac{1}{I} + \frac{1}{I^*}\right]$$

$$= E[L_{ED}^2].$$

Since I_{AM} is only approximately valid as opposed to I_{ED} which is exact, one might feel that the gain in sensitivity in using I_{AM} is only slight arising mainly due to the coverage probability being less than the specified level at times. A closer inspection will, however, show that this gain can be substantial. For example, $E(L_{AM}^2)/E(L_{ED}^2) \to 0$ as $\sigma_A^2 \to 0$ and $I, I^* \to \infty$.

7. An Illustrative Example

To illustrate our results, consider the following data on tread loss (in mils) after 20,000 miles for 4 brands of tires (Hicks [6], p. 52).

Brand

A	B	C	D
17	14	12	13
14	14	12	11
13	13	11	10
13	8	9	9

Here I = 4 and J = 4. We wish to obtain 95% prediction interval for $\bar{\bar{Y}}^*$ based on $I^* = 4$, $J^* = 2$. For constructing I_{ED}, we take $\ell_1 = \ell_2 = 1/2$ and $\ell_3 = \ell_4 = 0$. From the data, we get

$n_1 = 3$, $n_2 = 12$, $M_1 = 1/8$, $M_2 = 1/16$,

$\bar{\bar{Y}} = 12.06$, $S_1^2 = 10.23$, $S_2^2 = 4.19$,

$\hat{R} = 2.44$, $\hat{S}_p^2 = 4.19$, $\hat{N} \approx 4$.

The prediction intervals are:

I_{ED}: (9.55, 17.20), I_{AM}: (8.32, 15.80),

I_{AP}: (9.41, 14.71), I_{AS}: (8.61, 15.51).

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TABLE 1. Range of Estimated Coverage Probability

Specified Level = 1 - α = .95

I	J	I^*	J^*	I_{AP}	I_{AS}	I_{AM}
3	3	5	5	(85.2, 95.5)	(76.2, 97.9)	(92.9, 95.0)
		5	10	(85.4, 94.4)	(98.7, 99.5)	(92.1, 95.2)
		5	15	(85.3, 94.6)	(96.7, 99.5)	(91.0, 94.2)
		10	5	(83.4, 93.6)	(99.3, 99.7)	(92.8, 93.8)
		10	10	(85.3, 94.1)	(99.2, 99.7)	(93.8, 95.5)
		10	15	(82.9, 92.7)	(98.1, 99.8)	(92.4, 93.8)
		15	5	(84.7, 95.1)	(99.1, 99.9)	(93.6, 94.7)
		15	10	(85.5, 96.0)	(98.8, 99.7)	(94.4, 95.5)
		15	15	(83.9, 94.4)	(98.5, 99.7)	(94.0, 94.6)
3	5	5	10	(82.0, 91.3)	(98.2, 99.5)	(91.8, 93.0)
		5	15	(85.0, 93.7)	(97.7, 99.8)	(94.2, 95.6)
		10	10	(81.6, 91.7)	(99.0, 99.9)	(93.0, 93.9)
		10	15	(81.7, 92.3)	(97.9, 99.9)	(92.7, 94.3)
		15	10	(83.2, 92.9)	(98.3, 99.5)	(93.6, 94.0)
		15	15	(81.7, 92.7)	(98.6, 99.8)	(94.6, 95.7)
3	7	5	5	(81.4, 91.5)	(92.5, 94.1)	(94.2, 95.5)
		5	10	(82.6, 91.8)	(99.3, 99.9)	(94.0, 94.8)
		5	15	(84.2, 92.2)	(98.9, 99.7)	(93.9, 95.7)
		10	5	(82.5, 94.0)	(92.3, 94.7)	(94.3, 94.9)
		10	10	(81.9, 92.4)	(98.8, 99.8)	(93.9, 94.6)
		10	15	(83.3, 92.0)	(98.5, 99.9)	(94.4, 95.4)
		15	5	(82.5, 92.0)	(93.4, 94.5)	(94.4, 94.8)
		15	10	(82.9, 92.0)	(99.4, 100.0)	(95.2, 95.7)
		15	15	(82.2, 90.9)	(98.9, 99.7)	(94.7, 95.3)

Table 1 (Continued)

I	J	I^*	J^*	I_{AP}	I_{AS}	I_{AM}
5	3	5	5	(90.2, 94.3)	(97.7,100.0)	(93.6, 95.5)
		5	10	(89.1, 94.1)	(97.4, 99.7)	(91.7, 95.5)
		5	15	(89.9, 94.5)	(97.3, 99.5)	(91.1, 95.3)
		10	5	(89.7, 94.5)	(97.4,100.0)	(94.9, 95.9)
		10	10	(89.4, 94.4)	(96.4, 99.7)	(93.2, 94.6)
		10	15	(89.4, 94.0)	(96.6, 99.7)	(93.3, 95.1)
		15	5	(90.9, 94.6)	(97.2, 99.3)	(95.3, 95.9)
		15	10	(90.4, 94.4)	(96.9, 99.8)	(93.9, 95.7)
		15	15	(90.1, 95.5)	(97.2, 99.7)	(94.8, 96.2)
5	5	5	10	(88.1, 91.4)	(95.6, 99.9)	(92.3, 93.7)
		5	15	(87.8, 92.5)	(97.1, 99.6)	(93.0, 94.8)
		10	10	(90.2, 94.5)	(96.7, 99.5)	(94.2, 95.6)
		10	15	(90.5, 94.1)	(97.0, 99.4)	(94.7, 95.5)
		15	10	(88.3, 92.9)	(95.9, 98.9)	(93.6, 94.2)
		15	15	(90.8, 94.7)	(96.8, 99.8)	(95.1, 95.7)
5	7	5	5	(89.0, 93.2)	(95.0, 95.8)	(95.7, 96.2)
		5	10	(89.0, 92.5)	(97.7, 99.8)	(95.5, 96.3)
		5	15	(88.1, 90.7)	(95.7, 99.3)	(92.2, 93.6)
		10	5	(87.5, 92.0)	(94.0, 94.6)	(94.3, 95.0)
		10	10	(88.4, 92.2)	(96.3, 98.8)	(94.4, 94.8)
		10	15	(86.6, 91.5)	(96.7, 99.6)	(93.0, 94.9)
		15	5	(87.7, 92.4)	(94.4, 95.0)	(94.6, 95.3)
		15	10	(88.6, 92.7)	(96.5, 98.7)	(95.1, 95.4)
		15	15	(90.4, 93.6)	(97.7, 99.8)	(95.4, 96.2)
7	3	5	5	(91.7, 94.6)	(95.2, 99.6)	(93.0, 94.4)
		5	10	(89.7, 93.2)	(95.3, 99.9)	(91.9, 95.1)
		5	15	(89.2, 92.9)	(95.2, 99.7)	(89.7, 94.2)
		10	5	(92.0, 95.5)	(96.1, 99.4)	(94.2, 95.5)
		10	10	(90.2, 93.7)	(95.4, 99.4)	(93.0, 94.5)
		10	15	(91.3, 94.6)	(95.7, 99.7)	(92.5, 94.7)

Table 1 (Continued)

I	J	I*	J*	I_{AP}	I_{AS}	I_{AM}
		15	10	(90.4, 94.0)	(94.6, 99.4)	(92.8, 94.8)
		15	15	(90.6, 93.9)	(95.0, 99.7)	(92.1, 94.6)
7	5	5	10	(90.4, 93.2)	(95.5, 99.8)	(93.2, 94.6)
		5	15	(88.6, 92.1)	(95.6, 100.0)	(91.4, 94.9)
		10	10	(93.1, 95.3)	(96.3, 99.4)	(95.1, 95.7)
		10	15	(90.8, 93.7)	(96.8, 99.7)	(94.3, 96.1)
		15	10	(90.4, 94.2)	(95.1, 98.9)	(93.7, 94.3)
		15	15	(90.3, 94.3)	(95.5, 99.0)	(93.6, 94.6)
7	7	5	5	(88.8, 92.1)	(93.7, 94.2)	(94.0, 94.3)
		5	10	(91.1, 94.0)	(96.0, 98.8)	(95.0, 95.7)
		5	15	(92.5, 93.8)	(96.4, 99.4)	(94.4, 96.0)
		10	5	(90.7, 93.2)	(94.2, 94.5)	(94.4, 94.8)
		10	10	(91.4, 92.9)	(96.2, 97.6)	(95.5, 95.7)
		10	15	(88.8, 91.3)	(96.2, 98.9)	(94.9, 95.3)
		15	5	(90.7, 94.0)	(95.4, 95.6)	(95.4, 95.6)
		15	10	(90.3, 93.4)	(96.0, 98.2)	(95.4, 95.6)
		15	15	(90.3, 93.3)	(96.2, 98.7)	(94.5, 95.5)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
An exact and three approximate prediction intervals for the over-all mean of a future sample are obtained under a one-factor random model. The validity of these approximate intervals are assessed by a simulation study. A sensitivity comparison is also made.