Some Contributions by Purdue Industrial Engineers to Designed Experiments

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I. Background

Since the 1950's, Industrial Engineering (called General Engineering in the early days) has been one ot the "best" users of statistics. There has been a very good relationship between the statistics and industrial engineering faculty. In fact, Charlie Hicks was a part time faculty member in the Industrial Engineering School for awhile and at present a Quality Control course is split between the Statistics Department and Industrial Engineering. Much of the credit for this association goes to Professors Leimkuhler, Barany, Amrine, Greene, Lascoe, and Barash of the "mature" group.

Personally, it has been a very pleasant experience working with the Industrial Engineers over the years dating back to the early 1950's. I was looking over the booklet that Professor Amrine wrote on the "Roots and First Thirty Years" of Industrial Engineering at Purdue and was amazed to find that I knew so many of the 1985-86 Professors and graduate students over the last 30 years. It was rather informative to find out that I still know and have worked with so many of the young professors (including the ones who took classes from me a few years ago). The future looks very bright for Industrial Engineering at Purdue and much of this, I think, is due to its use of quantitative methods in such a variety of ways. One of these (which is a basis for acquiring data efficiently) is the area of design of experiments and I would like to share a few experiences in this area that I have had with some of the people in IE over the years.

II. Early Designs

In the Fall semester of 1964 an IE student by the name of Winston Charles Lister asked me a question that changed the whole way I looked at restrictions on randomization in designed experiments. Before this semester I had taught the design course (Stat 602) in a very conventional manner given in the books by Cochran and Cox (1957) and by Kempthorne (1952).

The question Lister asked was, "Why is the equation for a randomized complete block design (RCBD) the same as that for a two-factor factorial (one observation per cell) completely randomized design (CRD)?"

I stood there for what seemed like an eternity to me and thought, "This young man has exposed many possibilities in describing experiments in a quantitative sense". I really do not know how I answered the question but I definitely thanked him for exposing the subject so openly.

What had been written in the "good" textbooks such as Cochran and Cox (1957), and Kempthorne (1952) was that one should <u>not</u> test for blocks in a RCBD but the equation

looked exactly as if the design were CRD. In the "lesser" texts, blocks were tested using the interaction of blocks by treatment and no one (to my knowledge) in the statistical world made a comment about testing for blocks, possibly because the statisticians did not read these texts.

Anyway, for the next five years I tried to develop a way to put a restriction error in the RCBD equation and, to help all of this, another I.E. student by the name of Jon Robert Beeson (1965) came to my office in the Spring semester of 1965 and presented a problem for his thesis.

Beeson wanted to run a simulated study on heart valves. The set up was to include a tank of saline solution to simulate human blood, tubing to act as an artery that ran to and from a pump which simulated the human heart and a place in the tubing into which actual artificial cardiac valves were to be seated. The entire system was closed so that the saline solution could be pumped continuously through the tubing and holding tank once there was a heart valve seated in the tubing.

It was very easy to change the pumping rate to act as pulse rates but extremely difficult to change the valves similar to the difficulty a surgeon would have when placing an artificial heart in the aorta of a human being.

There were to be four heart valves and six pulse rates used in the experiment. The backpressure (an efficiency rating) of the valve was to be measured as well as some other dependent variables.

Beeson suggested the following design:

	Heart Valve (fixed)			
	2	4	1	3
Pulse Rates (fixed)	3	6	5	1
	1	2	3	4
	6	4	4	3
	4	5	6	2
	6	6	2	1
	2	2	3	6
	1	1	4	5
	5	3	1	3
	4	5	2	6
	3	4	5	4
	2	1	6	2
	5	3	1	5

where: the six pulse rates are repeated in the order above for each heart valve.

This arrangement allowed the research worker to seat valves only 4 times.

Beeson's suggested equation was:

$$y_{ijk} = \mu + V_i + P_j + VP_{ij} + \in_{(ij)k}$$

with corresponding degrees of freedom

$$48 = 1 + 3 + 5 + 15 + 24$$

and the mean square for $\hat{\in}_{(ij)k}$ was to be used as the denominator for all tests on Valves, Pulse Rates and the interaction of VP.

Of course, this was wrong because Heart Valves acted as blocks and blocks should not be tested in an RCBD. From books Beeson had read, and he showed one to me, his equation was justified.

How was I to make it clear to him that he must seat each valve type at least twice to find the correct variation estimate that would occur to evaluate value types? We contacted his major professor Jim Greene and the three of us agreed that he must run the experiment as follows:

Heart Valve (fixed)

Pulse rates (fixed)	1 Seating (random)		2 Seatings		3 Seatings		4 Seatings	
	4	7	1	3	2	6	5	8
	5	3	2	4	3	6	1	4
	2	1	3	2	1	5	3	6
	6	6	5	1	4	3	5	2
	1	2	1	6	5	2	2	5
	3	4	6	3	6	1	4	3
	4	5	4	5	2	4	6	1

where: #1 seating was for value type 2 and all six pulse rates run as indicated in that column. #2 seating was for value type 3 and so on.

The equation to analyze the data from this experiment is:

$$y_{ijk} = \mu + V_i + S_{(i)j} + P_k + VP_{ik} + SP_{(i)jk} + \in_{(ijk)}$$

and the corresponding degrees of freedom are:

$$48 = 1 + 3 + 4 + 5 + 15 + 20 + 0.$$

The test on Valve Types (V_i) uses the mean square for seatings in valve types ($S_{(i)j}$) and the P and VP uses the mean square for $SP_{(i)jk}$ as the denominator of their F's.

We were happy because there were only 8 seatings and there was a legitimate test on Valve Types and on Pulse Rates and the interaction, but how could I make experimenters aware that the concept of the first design was wrong?

Reflecting back on Lister's question the semester before, I introduced the restriction error, $\delta_{(i)}$, concept into the RCBD equation as follows:

$$y_{ij} = \mu + B_i + \delta_{(i)} + T_j + BT_{ij} + \in_{(ij)} .$$

When allowing $\delta_{(i)}$ to be representative of an error that would occur if the blocks were repeated, one could see that the error for blocks would be $\delta_{(i)}$ and not the interaction source. The basic concept is that if there is a restriction on randomization such as having all treatments appear in a block then one must recognize that to test for blocks requires a repeat of that block. If no repeat exists then one must place in the equation a term to represent that error and recognize there is no way to estimate the correct error variance for blocks without a repeat.

The first paper written on restriction errors was Anderson (1970). Dr. Gertrude Cox made a comment, after hearing this paper presented, that for the first time she understood why Dr. Cochran would not let her test for blocks when they wrote their book on "Experimental Designs". Quite a few papers have been written on this topic since then and they all came about because of an IE student's inquisitiveness.

Later the book by Anderson and McLean (1974) was published using many examples with restriction errors but the best example to this day is the cardiac valve example from another IE student.

III. Recent Designs

For years Cross Over designs have been used by animal scientists and pharmaceutical research workers to allow each individual (be they animals or humans) to be its own control. Usually in this design only two treatments or drugs are compared as follows:

	Sequence	
	1 Individuals	2 Individuals
Time	1 2 - - - 10	11 - - - 20
1	Drug A	Drug B
	Wash Out Period	
2	Drug B	Drug A

The equation for analyzing the data from such an experiment is:

$$y_{ijk} = \mu + S_i + I_{(i)j} + \delta_{(ij)} + T_k + ST_{ik} + IT_{(i)jk} + \in_{(ijk)}$$

with the corresponding degrees of freedom

$$40 = 1 + 1 + 18 + 0 + 1 + 1 + 18 + 0$$

where ST_{ik} = Drug (A or B) effect because of the Latin Square arrangement which assumes that Sequence $\times$ Time (ST) really is zero.

A picture of this is:

Sequence

Time	1	2
1	A	B
2	B	A

where the $\diagdown\!\diagup$ indicates the interaction of Sequence by Time which is exactly the main effect of A vs B; consequently the Drug effect (A vs B) is tested using the correct mean square IT.

A few years ago Professor Salvendy asked me how one should handle three treatments in a crossover design. In this case there are six sequences [instead of the two above $\left(\begin{smallmatrix} A & B \\ B & A \end{smallmatrix}\right)$]:

Sequences

Time	1	2	3	4	5	6
1	A	A	B	B	C	C
2	B	C	C	A	A	B
3	C	B	A	C	B	A

The design is better than the two sequence crossover because one can estimate a portion of the interaction of Sequence by Time (ST) after the sequence effect is removed. The reason for this is that $S \times T$ has 10 d.f. and only two d.f. are used to estimate the drug (A or B or C) effect leaving 8 d.f. for an estimate of part of the interaction.

Then, in about 1982, another IE student, Joe Sharit, who received his Ph.D. in 1984, brought the following complicated experiment to me:

In a study on attentional environments conducted in the Industrial Engineer at Purdue University, the following factors were considered:

	Factor	Levels	
a)	Information processing (P)	Lo	Hi
b)	Attentional environments (A)	Internal (I)	External (E)
c)	Paced Type (T)	Self (S)	Machine (M)

One task emphasized visual detection, labelled "external". The other task required mental solution to arithmetic problems and was called "internal". Each task took 10 minutes, consisting of 48 trials. One measurement on each variable was taken after all 48 trials were made per task. One task gave 1 observation per variable. For machine paced, definite periods were allotted for observing and making decisions; however for self paced the individual could use discretion in observing and deciding but was required to do the 48 trials per task. Many physiological variables were recorded.

Unfortunately in this type experiment the order that the treatment combinations (within the information processing levels) are presented to the subjects may have an influence on the measured variables. Hence one must take order into account. Within each information processing levels there are 4 treatment combinations of A and T or $4! = 24$ orders or sequences.

Of these 24 sequences the investigator was concerned with only 8:

1) $S_E M_E S_I M_I$ = Self external, Machine external, Self internal, Machine internal.
2) $M_E S_E S_I M_I$
3) $S_E M_E M_I S_I$
4) $M_E S_E M_I S_I$
5) $S_I M_I S_E M_E$
6) $M_I S_I S_E M_E$
7) $S_I M_I M_E S_E$
8) $M_I S_I M_E S_E$

The investigator was able to obtain 32 subjects to represent the population of operators who would be involved with this operation. He assigned 2 individuals to each order for both levels (lo, hi) of information processing. The layout (physical model) for the experiment was as follows:

			Information Processing (P)							
			Lo				Hi			
			Sequences (S)				Sequences (S)			
		(T)	1	2	···	8	1	2	···	8
		Paced	Indiv.(I)	Indiv.(I)	Indiv.(I)	Indiv.(I)	Indiv.(I)	Indiv.(I)	Indiv.(I)	Indiv.(I)
		Type	1 2	3 4	···	15 16	17 18	19 20	···	31 32
	I	S	S_E							M_I
Attentional Environments (A)		M	M_E							S_I
	E	S	S_I							M_E
		M	M_I							S_E

It follows that the correct model to analyze data from the experiment is (including df):

$$
\begin{array}{rcccccccccccc}
df & = & 1 & + & 1 & + & 7 & + & 7 & + & 16 & +0 & \\
y_{ijk\ell m} & = & \mu & + & P_i & + & S_j & + & PS_{ij} & + & I_{(ij)k} & + & \delta_{(ijk)} \\
 & & 1 & + & 1 & + & 7 & + & 7 & + & 16 & & \\
 & & A_\ell & + & PA_{i\ell} & + & SA_{j\ell} & + & PSA_{ij\ell} & + & IA_{(ij)k\ell} & & \\
 & & 1 & + & 1 & + & 7 & + & 7 & + & 16 & & \\
 & & T_m & + & PT_{im} & + & ST_{jm} & + & PST_{ijm} & + & IT_{(ij)km} & & \\
 & & 1 & + & 1 & + & 7 & + & 7 & + & 16 & & \\
 & & AT_{\ell m} & + & PAT_{i\ell m} & + & SAT_{j\ell m} & + & PSAT_{ij\ell m} & + & IAT_{(ij)k\ell m} & + & \in_{(i)k\ell m}
\end{array}
$$

All of this investigation on cross over designs with more than two treatments was the

basis for Chapter 8 written by Anderson and McLean in the book edited by Smolen and Ball (1984).

IV Summary

The Industrial Engineering School at Purdue University has been most stimulating in the development of new techniques used in the present day design and analysis of experiments.

V References

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