On A Sequential Subset Selection Procedure*

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ON A SEQUENTIAL SUBSET SELECTION PROCEDURE*

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Abstract

This paper deals with the problem of selecting the best population through the sequential subset selection approach. Based on the modified likelihood ratio of the probability density function of some invariant sufficient statistics, a sequential subset selection procedure is proposed. When the procedure terminates, one can assert with a guaranteed probability P^*, that the best population is included in the selected subset and that each selected population is within some fixed distance from the best population.

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key words: Sequential procedure, subset selection, likelihood ratio, best population, measure of separation.

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1. Introduction

Consider the problem of selecting the "best" among k populations. Suppose that observations can be obtained from the k populations sequentially. It is often desirable to terminate sampling from a population as soon as there is statistical evidence that it is not the best population, and this population is eliminated from further consideration. Selection through sequential comparison with elimination provides a significant advantage. To achieve a certain accuracy, it requires, on the average, substantially fewer samples than the fixed sample size procedures.

In sequential selection and ranking procedures, contributions have been made to select the best population by using the indifference zone approach. The simplest formulation of the indifference zone approach is the situation where one may wish to select only a single population and guarantee with a prespecified probability that the selected population is the best population provided some other condition on the parameters is satisfied, usually an indifference zone. However, in many real situations, it is hard or not always possible to specify the indifference (preference) zone condition. Thus, a reasonable and useful approach is to derive a sequential selection procedure to select a small subset containing the best population. However, it may happen that a poor population may be contained in the selected subset. Recently, Hsu (1981, 1982) and Hsu and Edwards (1983) studied methods to derive simultaneous upper confidence intervals for all measures of separation between the unknown best population and each (non-best) population under the location model. This motivates us to study selection rules such that, with some prespecified guaranteed probability, not only the best population is selected, but also, each selected population is very close to the best population.

In this paper, some sequential subset selection procedures achieving the goal described above are derived. These procedures are based on an invariant statistic for the parameters of interest. We consider observations from each pair of k populations and perform a modified sequential probability ratio test (MSPRT) based on the invariant statistics. This is done simultaneously for all pairs of populations and if a particular MSPRT terminates, then an appropriate population is removed from the set of contending populations. This is continued until only one population belongs to this set or some statistical evidence indicates that all the populations remaining in this set are within a (small) specified distance from the unknown best population. At each stage these procedures also provide some statistical inference about an upper bound on the measure of separation between the unknown best population and each remaining population.

2. Formulation of the Selection Problem

Let $\pi_1, \ldots, \pi_k$ represent $k (k \geq 2)$ populations and let X_{in} denote the n^{th} observation from population $\pi_i, i = 1, \ldots, k$. It is assumed that the observations $X_{in}, i = 1, \ldots, k; n = 1, 2, \ldots$ are independently distributed. Suppose that X_{in} has distribution function $F(x|\theta_i)$ depending on some unknown parameter θ_i for $i = 1 \ldots k$. Let $\underset{\sim}{\theta} = (\theta_1, \ldots, \theta_k)$ and let $\Omega = \{\underset{\sim}{\theta} | \underset{\sim}{\theta} = (\theta_1, \ldots, \theta_k)\}$ be the parameter space. For each i and j, let $\delta_{ij} = \delta(\theta_i, \theta_j)$ be a measure of separation between π_i and π_j where $\delta(\theta_i, \theta_j)$ as a function of θ_i and θ_j, is increasing (decreasing) in $\theta_i(\theta_j)$ when $\theta_j(\theta_i)$ is fixed, and satisfies the conditon that $\delta(\theta, \theta) = \delta_0$ for all θ. Define $\overline{\delta}_i = \min_{j \neq i}\{\delta_{ij}\}$ and $\overline{\delta} = \max_{1 \leq i \leq k} \overline{\delta}_i$. Population π_i is called the best population if π_i is the unique population such taht $\overline{\delta}_i = \overline{\delta}$. If more than one population has this property, one of them is tagged, and considered as the best population. We use (k) to denote the index of the best population and denote the best population by $\pi_{(k)}$.

Suppose that observations from the k populations are taken sequentially. The selection procedure will depend upon the observations through a sequence of statistics $\{T_{ij}(n), n \geq$

1}, which are defined to be functions

$$T_{ij}(n) = T_n(X_{i1}, \ldots, X_{in}; X_{j1}, \ldots, X_{jn}) \tag{2.1}$$

of the first n observations from populations π_i and π_j. In a given problem, the function T_n is chosen so as to indicate a measure of the separation between the populations in a reasonable way. Let $\tilde{T}_{ij}(n) = (T_{ij}(1), \ldots, T_{ij}(n))$. We assume that $\tilde{T}_{ij}(n)$ has a joint probability density $g_n(\tilde{t}_{ij}(n)|\delta_{ij})$ depending on the parameters θ_i and θ_j only through $\delta_{ij} = \delta(\theta_i, \theta_j)$. Usually, $T_{ij}(1), T_{ij}(2), \ldots$, are chosen so that it is both a sufficient and transitive sequence and also invariant sufficient for δ_{ij} (see Hall, Wijsman and Ghosh (1965)).

We assume that there is no information about the configuration of δ_{ij}'s, $1 \leq i, j \leq k, i \neq j$. However, we desire that each selected population should not be far from the best population. Let $\delta_{i(k)}$ denote the measure of separation from the population π_i to the best population $\pi_{(k)}$. Then, by our definition, $\delta_{i(k)} \leq \delta_0$. For a prespecified value $\delta_* < \delta_0$, population π_i is said to be good if $\delta_{i(k)} \geq \delta_*$ and bad otherwise. Let S denote the selected subset and $CS(\delta_*)$ denote the event that $\pi_{(k)} \in S$ and $\delta_{i(k)} \geq \delta_*$ for all $\pi_i \in S$. We desire a sequential subset selection procedure $\mathcal{P}$ such that

$$P_{\underset{\sim}{\theta}}\{CS(\delta_*)|\mathcal{P}\} \geq P^* \text{ for all } \underset{\sim}{\theta} \in \Omega, \tag{2.2}$$

where $P^*(k^{-1} < P^* < 1)$ is a prespecified probability level.

3. Sequential Selection Procedure $\mathcal{P}$

Let $h(\cdot)$ be a monotonically decreasing function such that $h(\delta_{ij}) = \delta_{ji}$. Let $\delta_*(< \delta_0)$ be a prespecified value used to specify the event $CS(\delta_*)$. Then $\delta_0 = h(\delta_0) < h(\delta_*)$. Let δ_1 be a value such that $\delta_0 < \delta_1 < h(\delta_*)$. Consider the likelihood ratio statistics

$$L_{ij}(n, a) = \frac{g_n(\tilde{T}_{ij}(n)|\delta_1)}{g_n(\tilde{T}_{ij}(n)|a)}, (n \geq n_0) \tag{3.1}$$

where $a \leq \delta_0$ and n_0 is some positive integer. Hoel (1971) and Gupta and Huang (1975) have used the statistics $L_{ij}(n,a), n \geq n_0$, to construct sequential selection procedures where n_0 is the initial sampling size of the procedures. For simplicity, we assume that $n_0 = 1$. We now define a sequential selection procedure $\mathcal{P}$ as follows:

Let $S_0 = \{\pi_1, \ldots, \pi_k\}$. For each $n \geq 1$, define

$$S_n = \{\pi_i \in S_{n-1} | L_{ji}(n, \delta_0) < \frac{k-1}{1-P^*} \text{ for all } \pi_j \in S_{n-1} - \{\pi_i\}\}. \tag{3.2}$$

That is, S_n is the set of contending popualtions up to stage n. At stage n, population $\pi_i \in S_n$ is labelled as good if $L_{ij}(n, \delta_*) \geq \frac{k-1}{1-P^*}$ for all $\pi_j \in S_n - \{\pi_i\}$. Let $|S_n|$ denote the size of the set S_n. The procedure terminates if either $|S_n| = 1$ or all the populations in S_n have been labelled as good. In either case, we take $S = S_n$; otherwise, we go to next stage. The procedure is thus continued.

4. Probability of a Correct Selection

Let $g_m(t|\tilde{t}(m-1), \delta)$ denote the conditional probability density function of $T_{ij}(m)$ given $\tilde{T}_{ij}(m-1) = \tilde{t}(m-1)$, and let $L_{ij}(n,a)$ be the statistic defined in (3.1). Then, the statistics $L_{ij}(n,a), n \geq 1$, can be rewritten as:

$$L_{ij}(n,a) = \frac{g_1(T_{ij}(1)|\delta_1)}{g_1(T_{ij}(1)|a)} \prod_{m=2}^{n} \frac{g_m(T_{ij}(m)|\tilde{T}_{ij}(m-1), \delta_1)}{g_m(T_{ij}(m)|\tilde{T}_{ij}(m-1), a)}, \tag{4.1}$$

where $\prod_{m=2}^{n}[\quad] = 1$ if $n = 1$. For each $n \geq 1$, let $\mathcal{F}_{ij}(n)$ denote the σ-field generated by $\tilde{T}_{ij}(n)$. Then,

Lemma 4.1. $\{L_{ij}(n, \delta_{ij}), P_{\underset{\sim}{\theta}}, \mathcal{F}_{ij}(n), n \geq 1\}$ forms a nonnegative martingale for $i \neq j$.

Proof: This lemma can be proved by a direct computation.

Now, let E and $E_i^c (1 \leq i \leq k, i \neq (k))$ be the events as defined below:

$$(4.2)\qquad \begin{cases} E = \{L_{i(k)}(n,\delta_{i(k)}) < \frac{k-1}{1-P^*} \text{ for all } \pi_i \in S_{n-1} - \{\pi_{(k)}\} \text{ for all } n \geq 1\}, \\ E_i^c = \{L_{i(k)}(n,\delta_{i(k)}) \geq \frac{k-1}{1-P^*} \text{ for some } n \geq 1\}. \end{cases}$$

Then, we have the following lemma:

Lemma 4.2. (a) $P_{\underset{\sim}{\theta}}\{E_i^c\} \leq \frac{1-P^*}{k-1}$ for all $i \neq (k), \underset{\sim}{\theta} \in \Omega$.

(b) $P_{\underset{\sim}{\theta}}\{E\} \geq P^*$ for all $\underset{\sim}{\theta} \in \Omega$.

Proof: Part (a) is a consequence of Lemma 4.1 and a lemma of Robbins and Siegmund (1973). For the proof of part (b), we have

$$P_{\underset{\sim}{\theta}}\{E\} \geq 1 - P_{\underset{\sim}{\theta}}\{\bigcup_{i\neq(k)} E_i^c\} \geq 1 - \sum_{i\neq(k)} P_{\underset{\sim}{\theta}}\{E_i^c\} \geq P^*.$$

This completes the proof of this lemma.

Now, for each $a \leq \delta_0$ (the value of a is chosen so that the joint probability density function $g_n(\tilde{T}_{ij}(n)|a)$ is well defined), let $A_{ij}(m,a) = \{L_{ij}(m,a) < \frac{k-1}{1-P^*}\}$. In the following, we also assume that the following condition is satisfied.

$$(4.3)\qquad \underline{\text{Condition A:}}\ \bigcap_{m=1}^{n} A_{ij}(m,b) \subset \bigcap_{m=1}^{n} A_{ij}(m,a) \text{ for all } n \geq 1 \text{ for } b \leq a \leq \delta_0.$$

The implication of (4.3) is that the values of the statistics $L_{ij}(n,a)$ for $n \geq 1$, never exceed the boundary level $\frac{k-1}{1-P^*}$ before that of the statistics $L_{ij}(n,b), n \geq 1$ when $b \leq a \leq \delta_0$. A sufficient condition for (4.3) to hold is that $A_{ij}(n,b) \subset A_{ij}(n,a)$ for all $n \geq 1$.

For each $n \geq 1, \pi_i, \pi_j \in S_{n-1}, i \neq j$, define $B_{ij}(n)$ and $D_{ij}(n)$ as follows:

$$(4.4)\qquad B_{ij}(n) = \left\{a \leq \delta_0 | L_{ij}(n,a) < \frac{k-1}{1-P^*}\right\},$$

$$(4.5)\qquad D_{ij}(n) = \begin{cases} \inf B_{ij}(n) & \text{if } B_{ij}(n) \neq \phi, \\ \delta_0 & \text{if } B_{ij}(n) = \phi, \end{cases}$$

where ϕ denotes the empty set. Also, let $D_{ii}(n) = \delta_0$.

Under Condition A, if $D_{ij}(n) < \delta_0$, then $L_{ij}(n,a) < \frac{k-1}{1-P^*}$ for all $D_{ij}(n) < a \leq \delta_0$ and $L_{ij}(n,b) \geq \frac{k-1}{1-P^*}$ for all $b < D_{ij}(n)$. For each $n \geq 1$, if $\pi_i \in S_{n-1}$, define

$$D_i(n) = \max_{1\leq m\leq n}\Big(\min_{\pi_j\in S_{m-1}} D_{ij}(m)\Big). \tag{4.6}$$

If $\pi_i \notin S_{n-1}$, let $n_i = \max\{m|\pi_i \in S_{m-1}\}$ and define $D_i(n) = D_i(n_i)$.

By definition of $D_i(n)$, for each $i = 1,\ldots,k, \{D_i(n)\}$ is an increasing sequence and bounded above by δ_0.

<u>Lemma 4.3</u>. Let $L_{ij}(n,a), S_n, D_i(n)$ and the event E be as defined in (3.1), (3.2), (4.6) and (4.2), respectively. Then, under Condition A,

$$E \subset \{\pi_{(k)} \in S \text{ and } \delta_{i(k)} \geq D_i(n) \text{ for all } \pi_i \in S_{n-1} \text{ for all } n \geq 1\}.$$

Proof: Since $\delta_{i(k)} \leq \delta_0$ for all i, then, under Condition A, we have

$$\begin{aligned}
E \equiv&\{L_{i(k)}(n,\delta_{i(k)}) < \frac{k-1}{1-P^*} \text{ for all } \pi_i \in S_{n-1} - \{\pi_{(k)}\} \text{ for all } n \geq 1\}\\
\subset&\{L_{i(k)}(n,\delta_0) < \frac{k-1}{1-P^*} \text{ and } \delta_{i(k)} \geq D_{i(k)}(n) \text{ for all } \pi_i \in S_{n-1} - \{\pi_{(k)}\}\\
&\qquad \text{for all } n \geq 1\}\\
\subset&\{\pi_{(k)} \in S \text{ and } \delta_{i(k)} \geq D_{i(k)}(n) \text{ for all } \pi_i \in S_{n-1} - \{\pi_{(k)}\} \text{ for all } n \geq 1\}\\
=&\{\pi_{(k)} \in S \text{ and } \delta_{i(k)} \geq D_{i(k)}(n) \text{ for all } \pi_i \in S_{n-1} \text{ for all } n \geq 1\} \qquad (4.7)\\
\subset&\{\pi_{(k)} \in S \text{ and } \delta_{i(k)} \geq \min_{\pi_j\in S_{n-1}} D_{ij}(n) \text{ for all } \pi_i \in S_{n-1} \text{ for all } n \geq 1\}\\
=&\{\pi_{(k)} \in S \text{ and } \delta_{i(k)} \geq D_i(n) \text{ for all } \pi_i \in S_{n-1} \text{ for all } n \geq 1\}.
\end{aligned}$$

An immediate consequence of Lemmas 4.2 and 4.3 is: Under Condition A,

$$P_{\underset{\sim}{\theta}}\{\pi_{(k)} \in S \text{ and } \delta_{i(k)} \geq D_i(n) \text{ for all } \pi_i \in S_{n-1} \text{ for all } n \geq 1\} \geq P^* \text{ for } \underset{\sim}{\theta} \in \Omega. \tag{4.8}$$

This result provides a sequential confidence region inference, with confidence level at least P^*, as follows: Simultaneously, at each stage n, the best population is not eliminated and the separation from each remaining population, say π_i, to the unknown best population is not less than $D_i(n)$ for all $n \geq 1$. Another consequence of Lemma 4.2 and Lemma 4.3 is that when the selection procedure $\mathcal{P}$ terminates, the event $CS(\delta_*)$ is guaranteed with probability at least P^*. We state this result as a theorem as follows:

Theorem 4.1. Let $\mathcal{P}$ be the sequential selection procedure defined in Section 3. Also, suppose that the Condition A in (4.3) holds. Then,

$$P_{\underset{\sim}{\theta}}\{CS(\delta_*)|\mathcal{P}\} \geq P^* \text{ for all } \underset{\sim}{\theta} \in \Omega,$$

provided that the procedure $\mathcal{P}$ terminates with probability one.

Proof: Note that when the selection procedure $\mathcal{P}$ terminates, then either $|S| = 1$ or all the populations in S must have been labelled as good at some stage. Let N be the stopping time of the selection procedure $\mathcal{P}$ and when $|S| \geq 2$, for each $\pi_i \in S$, let N_i denote the first time that π_i was labelled as good. Then, $L_{ij}(N_i, \delta_*) \geq \frac{k-1}{1-P^*}$ for all $\pi_j \in S_{N_i} - \{\pi_i\}$. Under Condition A, by definition of $D_{ij}(n), D_{ij}(N_i) \geq \delta_*$ for all $\pi_j \in S_{N_i} - \{\pi_i\}$ and thus, $D_{i(k)}(N_i) \geq \delta_*$ if $\pi_{(k)} \in S_{N_i} - \{\pi_i\}$. Also, note that $S = S_N$ and when $|S| \geq 2, N_i \leq N$ for all $\pi_i \in S$. Now from (4.7),

$$
\begin{aligned}
E \subset & \{\pi_{(k)} \in S \text{ and } \delta_{i(k)} \geq D_{i(k)}(n) \text{ for all } \pi_i \in S_{n-1} - \{\pi_{(k)}\} \text{ for all } n \geq 1\} \\
\subset & \{\pi_{(k)} \in S \text{ and } |S| = 1\} \cup \{\pi_{(k)} \in S, |S| \geq 2, \delta_{i(k)} \geq D_{i(k)}(N_i) \text{ for all } \pi_i \in S - \{\pi_{(k)}\}\} \\
\subset & \{\pi_{(k)} \in S \text{ and } |S| = 1\} \cup \{\pi_{(k)} \in S, |S| \geq 2, \delta_{i(k)} \geq \delta_* \text{ for all } \pi_i \in S - \{\pi_{(k)}\}\} \\
= & CS(\delta_*).
\end{aligned}
$$

Then, by Lemma 4.2, we have, for all $\underset{\sim}{\theta} \in \Omega$,

$$P_{\underset{\sim}{\theta}}\{CS(\delta_*)|\mathcal{P}\} \geq P_{\underset{\sim}{\theta}}\{E\} \geq P^*.$$

5. An Illustrative Example: Selecting the Population with the Largest Normal Mean

Let $\pi_1, \ldots, \pi_k$ be k populations and let X_{in} denote the n^{th} observation taken from population π_i. Assume that X_{in} has normal distribution with an unknown mean θ_i and a common known variance $\sigma^2 = 1, i = 1, \ldots, k$. Define the measure of separation between π_i and π_j as $\delta_{ij} = \theta_i - \theta_j$. Then, $\delta_0 = 0$ and $\overline{\delta} = \theta_{(k)} - \theta_{(k-1)}$ where $\theta_{(1)} \leq \ldots \leq \theta_{(k)}$ are the ordered parameters of θ_i's. Thus, the population with the largest mean is considered as the best population. For a given $\delta^* > 0, \pi_i$ is said to be good if $\theta_{(k)} - \theta_i \leq \delta^*$ and bad otherwise. For a prespecified probability $P^*(k^{-1} < P^* < 1)$, we wish to derive a sequential selection procedure such that

$$P_{\underset{\sim}{\theta}}\{\pi_{(k)} \in S \text{ and } \theta_{(k)} - \theta_i \leq \delta^* \text{ for all } \pi_i \in S\} \geq P^* \tag{5.1}$$

for all $\underset{\sim}{\theta} \in \Omega$.

For each $n \geq 1$, define $T_{ij}(n) = S_{in} - S_{jn}$, where $S_{in} = \sum_{m=1}^{n} X_{im}$. Let $\delta_* = -\delta^*$ and let $0 < \delta_1 < \delta^*$. Then,

$$\log L_{ij}(n, 0) = \frac{\delta_1}{2}(S_{in} - S_{jn}) - \frac{n\delta_1^2}{4}$$

and

$$\log L_{ij}(n, \delta_*) = \frac{\delta_1 + \delta^*}{2}(S_{in} - S_{jn}) + \frac{n(\delta^{*^2} - \delta_1^2)}{4}.$$

In order to apply the procedure $\mathcal{P}$ to this selection problem, we need to make sure that this procedure terminates with probability one.

<u>Lemma 5.1</u>. For the problem of selecting the population with the largest mean among k normal populations with a common known variance, the sequential selection procedure $\mathcal{P}$ terminates with probability one if $0 < \delta_1 < \frac{\delta^*}{2}$.

Proof: It suffices to show that for any two populations, say π_1 and π_2, with probability one, the event H, that either one of them will be eliminated (in comparison with the other) or both of them are labelled as good, occurs. Without loss of generality, we assume that $\theta_1 \geq \theta_2$.

First consider the case that $\theta_1 - \theta_2 > \frac{\delta_1}{2}$. Define $N_1 = \min\{n | L_{12}(n, 0) \geq \frac{k-1}{1-P^*}\}$. By the strong law of large numbers, $\frac{1}{n}$ log $L_{12}(n, 0) \longrightarrow \frac{\delta_1}{2}(\theta_1 - \theta_2 - \frac{\delta_1}{2}) > 0$ a.e. as $n \longrightarrow \infty$, while $\frac{1}{n}$ log $\frac{k-1}{1-P^*} \longrightarrow 0$ as $n \longrightarrow \infty$. Hence, $P_{\underset{\sim}{\theta}}\{N_1 < \infty\} = 1$.

Next, consider the case, $0 \leq \theta_1 - \theta_2 \leq \frac{\delta_1}{2}$. Define $N_{ij} = \min\{n | L_{ij}(n, \delta_*) \geq \frac{k-1}{1-P^*}\}$ for $i, j = 1, 2, i \neq j$, and $N_2 = \max(N_{12}, N_{21})$. By the strong law of large numbers again, $\frac{1}{n}$ log $L_{12}(n, \delta_*) \longrightarrow (\theta_1 - \theta_2 + \frac{\delta^* - \delta_1}{2})(\delta_1 + \delta^*)/2 > 0$ a.e. as $n \longrightarrow \infty$, and $\frac{1}{n}$ log $L_{21}(n, \delta_*) \longrightarrow (\theta_2 - \theta_1 + \frac{\delta^* - \delta_1}{2})(\delta_1 + \delta^*)/2 > 0$ a.e. as $n \longrightarrow \infty$. Hence, $P_{\underset{\sim}{\theta}}\{N_{ij} < \infty\} = 1$ for $i, j = 1, 2, i \neq j$ and so, $P_{\underset{\sim}{\theta}}\{N_2 < \infty\} = 1$.

Finally, one can observe that $\{N_1 < \infty\} \cup \{N_2 < \infty\} \subset H$. Thus, based on the above discussion, we have, $P_{\underset{\sim}{\theta}}\{H\} \geq P_{\underset{\sim}{\theta}}\{N_1 < \infty \text{ or } N_2 < \infty\} = 1$ for all $\underset{\sim}{\theta} \in \Omega$. Hence the proof of this lemma is complete.

Now, to guarantee the P^*-condition for the event $CS(\delta_*)$, from Theorem 4.1, it suffices to verify the Condition A given in (4.3). This can be easily verified.

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This paper deals with the problem of selecting the best population through the sequential subset selection approach. Based on the modified likelihood ratio of the probability density function of some invariant sufficient statistics, a sequential subset selection procedure is proposed. When the procedure terminates, one can assert with a guaranteed probability P*, that the best population is included in the selected subset and that each selected population is within some fixed distance from the best population.

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