COMPLETE MARKETS WITH DISCONTINUOUS SECURITY PRICE

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Abstract. A parameterized family of financial market models is presented. These models have jumps intrinsic to the price processes yet have strict completeness, equivalent martingale measures, and no arbitrage. For each value of the parameter $\beta(-2 \leq \beta < 0)$ the model is just as rich as the standard model using white noise (Brownian motion) and a drift; moreover as β increases to zero the model converges weakly to the standard model. A hedging result, analogous to the Karatzas-Ocone-Li theorem, is also presented.

Keywords: Market completeness, arbitrage, stochastic calculus, Azéma martingales, equivalent martingale measure, weak convergence, hedging strategies, Malliavin calculus, option pricing, Black-Scholes models, contingent claims, martingale central limit theorem.

1 Introduction

The standard model for continuous time capital asset pricing models assumes that asset prices follow diffusion processes with continuous sample paths (see, for example, [5]). An important consequence of this is that markets are "complete": that is, a contingent claim that is measurable with respect to the filtration generated by the stock price process is a redundant claim. (This means that there will always exist a self-financing trading strategy that replicates the contingent claim.) However as has been long known (see for example [16] or [9]), empirical studies show that stock prices often have jumps (that is, are not continuous). An interesting recent example concerning the term structure of interest rates and monetary policy, where jumps clearly occur, is given in [1].

Various models incorporating discontinuities have been proposed. The most common is the "jump diffusion" model, where Poisson jumps have been added to Brownian (or "white") noise, modeled by a Wiener process. This has many advantages: the noise process $W_t + N_t$ is still a Lévy process (a process with stationary and independent increments) and thus can be justified by central limit type arguments; the solution of the corresponding stochastic differential equation (the stock price process) is a strong Markov process. These models are useful to model stock prices whose jumps arise from exogenous events (such as natural disasters, interest rate announcements, etc.;

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see [16], [1], or [10]), rather than to model those for which the jumps are intrinsic to the "trading noise". However they do not have unique equivalent martingale measures, and thus do not naturally give a unique option price.

In attempts to allow for more general models, recent work has been devoted to studying what can be done when a model does not have completeness, and what adjustments can be made in those cases. See, for example, [15], [19], [20], [3].

In this paper we show that one can indeed have models that have market completeness and jumps that are intrinsic to the stock price with a unique equivalent martingale measure. To emphasize the intrinsic nature of the discontinuities in our model, we define two types of market completeness. Let Y_t be the stock price process, and let $\mathcal{F}_t = \sigma(Y_s; s \leq t)$ be its natural filtration, made right continuous and complete.

Definition 1.1 A model has *strict market completeness* if for any contingent claim $H \in L^1(\mathcal{F}_T)$ there exists a predictable process ξ_s^H such that

$$H = a + \int_0^T \xi_s^H dY_s,$$

where Y is the asset price process.

Definition 1.2 A model has *relaxed market completeness* if for any contingent claim $H \in L^1(\mathcal{F}_T)$ there exist predictable processes $(\xi_s^1, \dots, \xi_s^n)$, some n, and semimartingales $M^1, \dots, M^n$ such that $\sum_{i=1}^n M_t^i = Y_t$ and

$$H = a + \sum_{i=1}^n \int_0^T \xi_s^i dM_s^i,$$

where Y is the asset price process.

It is known (see [2, p.235] or [8, p.353]) that the only Lévy processes that have strict market completeness are Brownian motion alone and the Poisson process alone. Jeanblanc-Picqué and Pontier [10] treat the case where Y is the sum of a Wiener process and a Poisson process by using relaxed market completeness.

In this article, however, we produce a family of distinct semimartingales, indexed by a parameter $\beta(-2 \leq \beta \leq 0)$, that give rise to strict market completeness. The case $\beta = 0$ corresponds to the standard model of noise process W_t (a Wiener process) and a drift ("dt"); the case $-2 \leq \beta < 0$ gives rise to noises coming from compensated jump processes, plus drifts. The case $\beta = 0$ (the standard model) has noise processes that are Lévy of course; for the case $-2 \leq \beta < 0$ the noise processes no longer have independent increments (and thus are not Lévy), but nevertheless are strong Markov processes under the equivalent martingale measure. The freedom of choice of coefficients for the stochastic differential equation giving rise to the stock price process means that for each β we have a model as rich as the standard model. Moreover we show in Section 3 that these models converge to the standard model as β increases to 0. Consequently for very small β the sample paths will look as if they are Brownian, although occasional jumps will be noticeable; this seems to correspond to empirical observations.

In Section 2 we present our model, proving for it the existence of a unique equivalent martingale measure (and hence no arbitrage) and market completeness. In Section 3 we show the aforementioned weak convergence to the standard model. In Section 4 we calculate (at least in theory) a hedging strategy along the lines of Karatzas-Ocone-Li, who extended the Clark-Hausmann formula in the standard model case.

For unexplained stochastic calculus terms and notation we refer the reader to [18].

2 The Model

In the usual framework, a stock price is modelled as "geometric Brownian motion": that is, the unique solution of

$$Y_t = 1 + \int_0^t \sigma Y_s \, dW_s + \int_0^t mY_s \, ds \tag{2.1}$$

where W is a standard Wiener process. The process Y has a closed form expression:

$$Y_t = \exp\left(\sigma W_t + mt - \frac{1}{2}\sigma^2 t\right).$$

Now let X be a semimartingale and suppose

$$Y_t = 1 + \int_0^t \sigma Y_{s-} dX_s. \tag{2.2}$$

Then Y again has a closed form expression (cf, eg, [18, p.77]):

$$Y_t = \exp\left(\sigma X_t - \frac{1}{2}\sigma^2 [X,X]_t^c\right) \prod_{0<s\le t} (1+\Delta X_s)e^{-\Delta X_s},$$

where $\Delta X_s = X_s - X_{s-}$ denotes the jump at time s. Note that when $X_t = W_t + \frac{m}{\sigma}t$, (2.2) reduces to (2.1).

We are interested in conditions on X that lead to a complete market model on $[0,T]$, T a fixed time.

Theorem 2.1 *For each β, $-2 \le \beta < 0$, there exists a semimartingale $X = X^\beta$, satisfying*

(i) $X_t = L_t + \int_0^t J_s ds$ *where L is an L^2-martingale and $E\{\int_0^T J_s^2 d[X,X]_s\} < \infty$;*

(ii) $[X,X]_t = t + \beta \int_0^t X_{s-} dX_s$;

(iii) *furthermore X may be chosen so that for some $\varepsilon > 0$ and $c < \frac{\varepsilon}{|\beta|}$,*

(a) $1 + \beta X_{t-} J_t \ge \varepsilon > 0$,

(b) $|X_{t-}J_t| \leq c$.

For such an X there exists a unique equivalent martingale measure P^* making X a bounded martingale. Moreover if Y given in (2.2) is the security price, then the financial market model is complete: that is, every bounded contingent claim $H \in (\Omega, \mathcal{F}(X)_T)$ admits a representation

$$H = H_0 + \int_0^T \xi_s^H dY_s.$$

Proof: First suppose there exists an X satisfying (i), (ii) with ε as in the statement of the theorem. Define

$$K_s = \frac{-J_s}{1+\beta X_{s-}J_s}.$$

From (i) we have $[X,X]_t = [L,L]_t$, so

$$\begin{aligned} E\left\{\left(\int_0^t K_s dL_s\right)^2\right\} &= E\int_0^t K_s^2\, d[L,L]_s \\ &= E\int_0^t \frac{J_s^2}{(1+X_{s-}J_s)^2}\, d[X,X]_s \\ &\leq E\frac{1}{\varepsilon^2}\int_0^t J_s^2\, d[X,X]_s < \infty \end{aligned}$$

by (i). Therefore $\int_0^t K_s dL_s$ is well defined.

Next define

$$Q_t = 1 + \int_0^t Q_{s-}K_s\, dL_s$$

and

$$dP^* = Q_T dP.$$

We need to show P^* defines a true probability measure. By (i) we have $\Delta L_t = \Delta X_t$, and by (ii) we have $(\Delta L_t)^2 = (\Delta X_t)^2 = \beta X_{t-}\Delta X_t$, whence $\Delta L_t = \Delta X_t = \beta X_{t-}$. Thus $\Delta\int K_s dL_s = K_t\Delta L_t = \frac{-J_t}{(1+\beta X_{t-}J_t)}\beta X_{t-}$, and in absolute value this is less than or equal to $\frac{|\beta|c}{\varepsilon} < 1$ by (iii).

We also need to verify that $E\{Q_T\} = 1$. To do this it suffices to show that $\langle L, L\rangle$ is bounded a.s. (see, eg, [15, p.158]). Since $[L,L] = [X,X]$, we have

$$[L,L]_t = t + \beta\int_0^t X_{s-}\, dX_s = t + \beta\int_0^t X_{s-}\, dL_s + \beta\int_0^t X_{s-}J_s\, ds$$

which implies that if

$$N_t = t + \beta\int_0^t X_{s-}J_s\, ds,$$

then $[L,L]_t - N_t$ is a martingale, and if N is nondecreasing, then $N = \langle L, L\rangle$. However $\frac{dN}{dt} = 1 + \beta X_{t-}J_t \geq \varepsilon > 0$, so $N = \langle L, L\rangle$; that is,

$$\langle L, L\rangle_t = t + \beta\int_0^t X_{s-}J_s ds.$$

Since $|X_{-}J| \leq c$, we have that $\langle L, L\rangle$ is bounded a.s. on $[0,T]$.

Thus P^* is a true probability measure.

It remains to show that X is a bounded martingale under P^*. By Girsanov's theorem (see [4, p.238]) we have that $\widetilde{L}$ is a local martingale under P^*, where

$$\begin{aligned}
\widetilde{L}_t &= L_t - \int_0^t \frac{1}{Q_{s-}}\, d\langle Q, L\rangle_s \\
&= L_t - \int_0^t \frac{1}{Q_{s-}} Q_{s-} K_s \, d\langle L, L\rangle_s \\
&= L_t - \int_0^t K_s (1 + X_{s-} J_s)\, ds \\
&= L_t + \int_0^t J_s \, ds \\
&= X_t.
\end{aligned}$$

Moreover, we have that X satisfies (ii) under P^* as well, since (ii) holds path by path a.s. (dP), and P and P^* are equivalent. Emery [6] has shown that for each β, $-2 \leq \beta \leq 0$, there is a unique local martingale solution of (ii), which he has called the Azéma martingales. The uniqueness of this solution implies the uniqueness of P^*. Moreover such a solution X is bounded and thus a true martingale. Finally, Emery has further shown that such an X has the martingale representation property and we use that fact to establish market completeness.

Next consider the stock price Y, and let H be a bounded contingent claim, $\mathcal{F}_T$-measurable. Then we know there exists a predictable process φ_s such that

$$H = E^*(H) + \int_0^T \varphi_s \, dX_s.$$

For $\xi_s = \varphi_s \frac{1}{\sigma Y_{s-}}$, and then we have

$$H = E^*(H) + \int_0^T \left(\varphi_s \frac{1}{\sigma Y_{s-}}\right) dY_s = E^*(H) + \int_0^T \xi_s \, dY_s$$

which means that H is a redundant claim, ξ_s is a self-financing strategy that replicates the claim H, and the fair (Black-Scholes type) price of H is $E^*(H) = E(HQ_T)$.

Finally it remains to show that for each β, $-2 \leq \beta \leq 0$, such an X as specified in the hypotheses actually exists(!). Note that for $\beta = 0$, we can take $L = W$, the Wiener process and $J = m/\sigma > 0$, the constant process. Otherwise, as was noted above, there exists a (weakly) unique solution of

$$[M, M]_t = t + \beta \int_0^t M_{s-}\, dM_s \tag{2.3}$$

where $-2 \leq \beta < 0$ and M is required to be a local martingale. Emery [6] calls this the *structure equation,* and has in fact shown existence and (weak) uniqueness of solutions for all β. Moreover (for $-2 \leq \beta < 0$), he has shown that $[M,M]^c = 0$ and that M is bounded:

$$M_t^2 \leq M_0^2 + \frac{-2}{\beta}t.$$

It is clear that $\langle M, M\rangle_t = t$, as well.

Let X denote the solution of (2.3) under a Probability law we will call P^*. Let J be predictable such that $\int_0^t J_s^2 ds \leq c < \infty$ a.s., and further require that $J_t \Delta X_t > -1$; that is, $\Delta X_t > \frac{-1}{J_t}$, when $|\Delta X_t| > 0$. (Note that since $X_t^2 \leq X_0^2 + \frac{2}{-\beta}t$, we have $|\Delta X_t| \leq 2\sqrt{X_0^2 + \frac{2}{-\beta}t}$, so any J_t such that $|J_t| < \frac{1}{2\sqrt{X_0^2+\frac{2}{-\beta}t}}$ suffices.) Then if R is the unique solution of

$$R_t = 1 + \int_0^t R_{s-} J_s dX_s,$$

we have that R is a positive uniformly integrable martingale (because $\int_0^t J_s^2 ds$ is bounded), and thus $dP = R_T dP^*$ is a true probability law. Girsanov's theorem yields

$$L_t = X_t - \int_0^t \frac{1}{R_{s-}} d\langle R, X\rangle_s$$

is a P-martingale. But

$$L_t = X_t - \int_0^t \frac{1}{R_{s-}} R_{s-} J_s d\langle X, X\rangle_s = X_t - \int_0^t J_s ds,$$

whence $X_t = L_t + \int_0^t J_s ds$ is the sought after semimartingale. Since X satisfies (2.3), we have that X satisfies (i) and (ii) under P.

Note that since $|X_t| \leq \sqrt{X_0^2 + \frac{2}{-\beta}t}$, if we take $|J_t| \lneqq \frac{c}{\sqrt{X_0^2+\frac{2}{-\beta}t}}$, and also

$$\text{sign}(J_t) = -\text{sign}(L_{t-} + \int_0^{t-} J_s ds) = -\text{sign}(X_{t-}),$$

then we have both $|X_{t-}J_t| \leq c$ and $1 + \beta X_{t-}J_t \gneqq 0$. In fact we have $1 + \beta X_{t-}J_t \geq 1$, $0 \leq t \leq T_0$, and so we can take $\varepsilon = 1$, and c any constant less than $1/|\beta|$. Any such J, even a deterministic one except for the requirement on the sign, gives a solution of (i), (ii), and (iii) for each β, $-2 \leq \beta < 0$. ■

Comment: A consequence of Theorem 2.1 is that these models have no arbitrage opportunities. Indeed under P^*, X is a bounded martingale. Therefore Y is a local martingale. However since $\langle X, X\rangle_t = t$, we know that Y is a true martingale (see, e.g., [17, p.158]). It is then a standard result (e.g., [13, p.6]) that there are no arbitrage opportunities.

3 Convergence to the Standard Model

In Section 2 we showed that for each β, $-2 \leq \beta < 0$, there exists a non-Poissonian complete market model with jumps. Let P^*_β denote the equivalent martingale measure for a process X^β, satisfying the conditions of Theorem 2.1. Then $E^*_\beta(H)$ is the price of a contingent claim. As β increases to 0, the process X^β has more and more compensated jumps, and the paths converge to the paths of the standard model (with Brownian, or Wiener, market noise). In this section we make sense of the intuitive notion that $\lim_{\beta \uparrow 0} E^*_\beta(H) = E_W(H)$, where E_W denotes expectation under the Wiener measure.

Let $M = M^\beta$ be the unique martingale solution of

$$[M, M]_t = t + \beta \int_0^t M_{s-} d M_s$$

for $-2 \leq \beta < 0$. (Uniqueness is in distribution.)

Theorem 3.1 *M^β converges weakly to W, the standard Wiener process, as β increases to 0.*

Proof: We need only to verify sufficient conditions of a martingale central limit theorem. Writing M for M^β, first note that

$$(\Delta M_t)^2 = \Delta[M, M]_t = \beta M_{t-} \Delta M_t,$$

thus

$$\begin{aligned} E\left\{\sup_{t \leq T} (\Delta M_t)^4\right\} &= E\left\{\sup_{t \leq T} |\beta M_{t-} \Delta M_t|^2\right\} \\ &= E\left\{\sup_{t \leq T} \Delta \left(\int_0^t \beta M_{s-}\, dM_s\right)^2\right\} \\ &\leq E\left\{\left(\int_0^T \beta M_{s-}\, dM_s\right)^2\right\} \\ &= E\left\{\int_0^T (\beta M_{s-})^2 d[M, M]_s\right\} \\ &= \beta^2 E \int_0^T (M_{s-})^2 ds \\ &= \beta^2 \int_0^T E(M_{s-}^2)\, ds \\ &= \beta^2 \int_0^T E\{\langle M, M \rangle_s\}\, ds \\ &= \beta^2 \int_0^T s\, ds = \frac{\beta^2 T^2}{2} \end{aligned} \tag{3.1}$$

which converges to 0 as β tends to 0.

Also, we have

$$E\left\{\left([M^\beta, M^\beta]_t - t\right)^2\right\} = E\left\{\left(\int_0^t \beta M^\beta_{s-} dM^\beta_s\right)^2\right\} \tag{3.2}$$
$$= \beta^2 E \int_0^t M^2_{s-}\, d[M, M]_s$$
$$= \beta^2 E \int_0^t M^2_{s-}\, ds$$
$$= \beta^2 \int_0^t s\, ds = \frac{\beta^2 t^2}{2} \le \frac{\beta^2 T^2}{2}$$

which also tends to 0 as β tends to 0. Combining (3.1) and (3.2) we have that M^β satisfies condition (a), p. 340, in [7], of the Martingale Central Limit Theorem. ∎

In Section 2 we introduced the security price process Y:

$$Y_t = 1 + \int_0^t \sigma Y_{s-} dX_s,$$

where $X = M^\beta$ under P^*. More generally, we can let

$$dY_t^\beta = f(t, Y_{t-}^\beta) dX_t^\beta$$

with f continuous such that the solution Y^β is unique. Typically a contingent claim is of the form $H^\beta = g(Y_T^\beta)$, or more generally

$$H^\beta = G(Y_\bullet^\beta),$$

where G is a functional on the space of paths which are right continuous with left limits on $[0, T]$.

The next corollary shows that the price of the contingent claim for $\beta < 0$ tends to the price of the contingent claim of the standard models as β tends to 0. If β is close to zero, and thus the stock price process has sample paths that look as though they are Brownian with occasional jumps, then the price of the contingent claim under the model proposed here will be close to the price for the standard model.

Corollary 3.1 *If the contingent claim H is of the form $H^\beta = g(Y_T^\beta)$ where g is continuous, or of the form $H^\beta = G(Y_\bullet^\beta)$ where G is continuous in the uniform norm, then $\lim_{\beta \uparrow 0} E^*_\beta(H) = E^*_W(H)$, where P^*_W is the Wiener measure and thus $H = g(Y_T)$ or respectively $H = G(Y_\bullet)$), and*

$$dY_t = f(t, Y_t) dW_t.$$

Proof: Since $\sup_{-2<\beta\le 0} E\{[M^\beta, M^\beta]_T\} = T < \infty$, we have that $(M^\beta)_{-2\le\beta\le 0}$ trivially satisfies UCV (as defined, eg, in [12, p.23]) and hence Y^β converges weakly to Y by Theorem 3.1 combined with, e.g., Theorem 3.6 of [12, p.33]. Since g is continuous and G is continuous in the uniform norm, the result follows. ∎

4 The Hedging Strategy

In the standard model, if the claim H is of the form $H = g(Y_T)$, one is able to use the Markov property of Y along with known properties and distributions related to Brownian motion to calculate explicitly a hedging strategy. In the case of more complicated contingent claims, as when H is a functional of the Brownian paths known as look-back options, one cannot in general find an explicit formula for the hedging strategy. In some special cases, such as the maximum option, closed form solutions exist, but these depend heavily on the rich structure of Brownian motion. Indeed, the general result is as follows: if the contingent claim has the form $H = F(W)$, under technical hypotheses on F one has

$$H = E^*(H) + \int_0^T E^*\{D_sF(W) \mid \mathcal{F}_s\}\, dW_s \tag{4.1}$$

where $D_sF(W)$ denotes the Malliavin derivative process of $F(W)$ (see [11] for details). It is this formula (4.1) due to Karatzas, Ocone, and Li that we will extend here.

Let us fix β, $-2 \leq \beta < 0$. Emery [6] has shown that the Azéma martingale $M = M^\beta$ has the Chaos Representation Property (CRP). Let Σ_n be the increasing simplex on $\mathbb{R}^n_+$

$$\Sigma_n = \{(t_1, \ldots, t_n) \in \mathbb{R}^n_+ : 0 < t_1 < \ldots < t_n\}$$

and extend f defined on Σ_n by making f *symmetric* on $\mathbb{R}^n_+$. Define

$$I_n(f) = n! \int_{\Sigma_n} f(t_1, \ldots, t_n) dM_{t_1} \ldots dM_{t_n}.$$

Let $\mathcal{H}_n = \{I_n(f); f \in L^2(\Sigma_n)\}$, where L^2 is under Lebesgue measure. We define

$$\mathbb{D}_{1,2} = \left\{ H = \sum_{n=0}^{\infty} I_n(f_n) : \sum_{n=0}^{\infty} n n! \|f_n\|_n^2 < \infty \right\}.$$

For $H \in \mathbb{D}_{1,2}$, define $D_tH = \sum_{n=0}^{\infty} n\, I_{n-1}(f_n(\cdot, t))$. We need the following from [14, Theorem 4.5]:

Theorem 4.1 *Let $H \in L^2(dP)$ and also $H \in \mathbb{D}_{1,2}$. Then*

$$H = E(H) + \int_0^1 {}^p(D_tH)\, dM_t,$$

where ${}^p(D_tH)$ denotes the predictable projection of the process $(D_tH)_{t\geq 0}$. (The reader may consult *[14]* for more details on Chaos Representation.)

Comment: We could have written $E\{D_tH \mid \mathcal{F}_t\}$ for the predictable projection ${}^p(D_tH)$ to emphasize the analogy with the Karatzas-Ocone-Li theorem, but this would be a little sloppy in our framework for technical reasons: such a notation could easily be interpreted as the optional projection rather than the predictable projection, and it is the latter that we need. (In the Brownian case the optional and predictable projections coincide, but they are different in general for martingales with jumps.)

Theorem 4.2 *For $-2 \le \beta < 0$, let $X = X^\beta$ be a semimartingale satisfying the conditions of Theorem 2.1. Let $Y_t = 1 + \int_0^t \sigma Y_{s-}\, dX_s$ be the security price process. Let H be a contingent claim such that $H \in \mathbb{D}_{1,2}$ under P^*, the equivalent martingale measure. Then*

$$H = E^*(H) + \int_0^T \frac{1}{\sigma Y_{s-}} {}^p(D_s H)\, dY_s; \tag{4.2}$$

that is, $\frac{1}{\sigma Y_{t-}} {}^p(D_t H)$ is the hedging strategy.

Proof: Under P^* we have, by Theorem 4.1, that $H = E^*(H) + \int_0^T {}^p(D_s H)\, dX_s$, and thus (4.2) follows trivially (P^*). Since P is equivalent to P^*, it holds for P as well. ■

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