BOOK REVIEW OF
ARBITRAGE THEORY IN CONTINUOUS TIMES
BY TOMAS BJÖRK
OXFORD UNIVERSITY PRESS, 1999

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Abstract

We review the new book on Financial Derivatives by Tomas Björk. The review was requested by the Journal of Finance.

Are you planning to teach a master's level course on Mathematical Finance? Tomas Björk may have written a textbook for you. Since to be done properly (by a mathematician's standard) such a course would need to be taught at the PhD level, we are discussing a level where compromises and choices need to be made. (This is of course due to the advanced nature of the subject matter, which requires measure theoretic probability theory, stochastic (Itô) calculus, some partial differential equations, and some (stochastic) numerical analysis as prerequisites (!).) Different approaches to this blend already exist; up until this book my two favorites were Baxter and Rennie [1] (whose approach is spare and basic and strictly heuristic, but nevertheless careful and correct as far as it goes), and Lamberton and Lapeyre [2] (whose approach is at the more advanced end and rigorous, but stops short of treating the more advanced topics).

Björk makes many choices and compromises and reasonable people can disagree about their merits. For example, I think he relies on partial differential equations too much – by concentrating on European style options where they play a more central role – and thus he gives a false impression that PDE techniques are more useful than they actually are. In addition he completely avoids discussion of numerical issues, making his book more theoretical than applied. However, he really does succeed in imparting understanding and a feeling for the subject: This book is a successful, good book.

Economics is a subject that begs to be made easy with mathematics. Subjects such as "rates of change" that are transparent using calculus, get translated into maddening long and confusing paragraphs when explained to students innocent of the marvels of differential calculus. So it is as well with Finance theory and stochastic calculus. The two Fundamental Theorems of Asset Pricing: that (1) the existence of an equivalent martingale measure [risk neutral measure] is equivalent to the absence of arbitrage and (2) that the uniqueness of such a measure is equivalent to market completeness, are central to a profound (theoretical) understanding of the subject. At this level they need not be proved, but to explain them and to use them one needs Girsanov's theorem from stochastic calculus theory; Björk decided not to go this route, and the result is an occasional morass of verbiage that this reviewer (at least) found confusing.

The above said, since one cannot do all in one book, let us look at the many successes in Björk's book. Björk's discussion of Incomplete Markets (Chapter 10) is brilliant: by leaving the domain of the stock market to consider insurance on the weather for a proposed vacation, he lucidly explains why markets can be naturally and unavoidably incomplete. Chapter 8, "Parity Relations and Delta Hedging" addresses the topic of hedging using options and a "buy–and–hold portfolio." This topic is rarely treated in this type of book and is sufficiently well done here that one wishes the chapter were more complete. The end of the book (Chapters 16-18) give a really lovely introduction to the difficult topic of interest rate models. The standard models are presented (Vasicek, Cox-Ingersoll-Ross, etc.), and while they are each in turn briefly treated, there is no discussion as to which might be better or why – for example – one has a square root in the CIR model. (There is no reason based on economics; it is simply a model that gives desired properties. However

this point should be made.) More importantly the Heath–Jarrow–Morton models are nicely treated.

Publishers of trade books have abandoned the traditional service of copy editing books, and it is now up to authors to check their own work. A copy editor might have noticed that SEK (Swedish Krona) is defined on page 1 and is not in the index [there is no notation index] and woe to the reader who skips the introduction; that LIBOR is defined as the simple forward rate (p. 230) but that the acronym is never explained to the curious and economics innocent (London Inter Bank Offer Rate), and its reference in the index is vague at best; and that there are misprints such as saying that the increment of a Wiener process $W_t - W_s$ has a Gaussian $N(0, \sqrt{t-s})$ distribution (page 27) instead of the more common notation $N(0, t-s)$. (There are of course also several misprints such as "optimal" on page 227.) One last issue (and this is quite subjective): this reviewer finds objectionable a typography of the form

Question:

Who chooses the martingale measure?

Answer:

The market!

(page 150). Such large boxed slogans appear a few times in this book, to its detriment I am afraid.

But let us not end on such a petty note. This book strikes a nice compromise between mathematical rigor and heuristics. It focuses on European type options of the form $f(S_T)$ where S is a price process, and it relies heavily on partial differential equation techniques to study them. Numerical issues are not treated. Interest rates are given an especially nice treatment. Lucid and original explanations appear consistently throughout the book, making it one of the best choices available today for a master's level introductory course to Finance theory.

REFERENCES

1. M. Baxter and A. Rennie, *Financial Calculus*, Cambridge University Press, 1996.
2. D. Lamberton and B. Lapeyre, *Introduction to Stochastic Calculus Applied to Finance*, Chapman and Hall, 1996.